IJFANS INTERNATIONAL JOURNAL OF FOOD AND NUTRITIONAL SCIENCES

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 10,Oct 2022

Deep Learning Galerkin Method for Solving Ordinary Differential Equations

Raghvendra Singh[1], Vikashdeep Yadav[2], Purvi J Naik[3], Pushpendra Kumar[4], Rajendra Kumar Tripathi[5*]

[1]School of Sciences, UP Rajarshi Tandon Open University, Prayagraj

[2]Department of Mathematics, Dronacharya Govt. College, Gurugram (Haryana)

[3]Department of Mathematics, Science and Humanities, UPL University of Sustainable Technology, Bharuch, Gujarat

[4]Department of Mathematics, Shri Khushal Das University, Pilibangan, Hanumangarh

[5]Applied science and Humanities (Mathematics), Faculty of Engineering and Technology, Khwaja Moinuddin chishti Language university, Lucknow

Email: 10singh.raghvendra@gmail.com; yadav.vikasdeep89@gmail.com; naikpurvi2004@yahoo.co.in; pushpendrasaini22@gmail.com; drrktripathi_fgiet@rediffmail.com

Corresponding author*

Abstract:

To approximate the numerical answer of linear second-order regular differential equations with combined boundary situations, we recommend a Deep Learning primarily based totally Galerkin Method primarily based totally on deep neural community getting to know algorithms. Deep getting to know is blended with the Galerkin Method on this method. Instead of mixing linear foundation functions, deep neural networks are used withinside the proposed work. To fulfill the differential operators and boundary situations, we use the gradient descent set of rules to teach the neural community mesh-unfastened with out meshing. Furthermore, the convergence of the loss characteristic and the convergence of the neural community to the precise answer withinside the L2 (Eulidean Distance) norm below sure situations show the approximate cappotential of a neural community. Last however now no longer least, a few numerical experiments display the neural networks' intuitive cappotential to approximate.

Keywords: Differential Equation, Neural Network, Boundary Condition, Galerkin Method.

Introduction:

In the Differential Equations (DE) may explain mathematical models in a variety of disciplines. Since the inception of calculus theory, DEs have been utilized to explain a variety of natural phenomena and applied to several scientific and industrial applications. High-dimensional differential equations are used withinside the fields of physics, engineering, and aerospace,

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 10,Oct 2022

amongst others. Unfortunately, the answers of the bulk of DEs can not be represented withinside the shape of analytic answers, for this reason their numerical answers are in particular significant. Although numerous numerical approaches, together with the finite distinction method, finite detail method, and finite quantity method, had been evolved up to now for fixing DEs, those techniques nonetheless have a few limitations [1-3]. Concerning troubles of extra dimensionality, the computing value of the boom in grid factors will increase exponentially with dimension. Consequently, locating numerical answers has been a problem for a completely lengthy time.

Literature Survey:

The deep neural network model has demonstrated great success in artificial intelligence as a result of the enormous development of accessible data and computational resources. Recent research suggests that neural networks may be used to solve DEs. Deep neural networks with several layers do remarkably well at representing complex datasets. Moreover, [4-7] proposes a variety of efficient techniques for solving high-dimensional DEs. In the majority of these studies, only numerical calculations and illustrations of the validity of numerical solutions are presented.

A short assessment of Deep Convolution Neural Networks and the consequences in their experiment. They have carried out Alex Net on Dell Pentium processor the usage of MATLAB deep studying toolbox. They have labeled 3 picture datasets. The first dataset carries 4 hundred photos of kinds of animals that had been labeled with 99.1 percentage accuracy. The 2nd dataset carries 4 thousand photos of 5 kinds of plant life that had been labeled with 86.sixty four percentage accuracy. In the primary and 2nd datasets, seventy percentage of randomly selected samples from every magnificence had been used for education. The 0.33 dataset carries 40 photos of stained pleura tissues from rat lungs are labeled into training with seventy five percentage accuracy. In this records set 80 percentage of randomly selected samples had been utilized in education the model.

Here, based on the architecture of, we directly use the deep Galerkin Method to solve second-order differential equations without using the Monte Carlo Method [8-12]. This approach is distinct from the classic Galerkin Method since it combines the Galerkin Method with deep learning. The deep Galerkin Method substitutes the linear combination of basis functions with a deep neural network. We train the neural network using a random sample of space points and

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 10,Oct 2022

the gradient descent approach to fulfill the differential operators and boundary constraints. We do not need to create a grid for this procedure. The deep Galerkin Method can solve high-dimensional DEs for this reason as well. The provided approach is far easier and more successful. In addition, we derive the convergence of both the loss function and the neural network. Finally, numerical experiments illustrate the performance of the strategy. A new discussion about optimization techniques like soft computing, fuzzy logic and managerment techniques like inventory and supply chain management by (24-56).

The 3 adaptive strategies to enhance the computational overall performance of deep neural network (DNN) strategies for high-dimensional differential equations (DEs). They are the adaptive desire of the loss feature, adaptive activation feature, and adaptive sampling, all of with a purpose to be implemented. They are the adaptive desire of the loss feature, adaptive activation feature, and adaptive sampling, all of with a purpose to be implemented to the schooling manner of a DNN for Des. Several numerical experiments have proven that their adaptive strategies substantially enhance the computational accuracy and boost up the convergence pace mean “without”. Several numerical experiments have proven that their adaptive strategies substantially enhance the computational accuracy and boost up the convergence pace with out a want to growth the variety of layers or the variety of neurons of a DNN.

In this part, diverse researchers gift their outcomes and strategies, increasing at the previous commonly used. In this part, diverse researchers gift their outcomes and strategies, increasing at the previous have a look at of a layout and implementation studies at the Galerkin Method. A new iterative energy and amplitude correction (IPAC) set of rules to simulate nonstationary and non-Gaussian techniques. The proposed set of rules is rooted withinside the idea of defining the stochastic techniques withinside the remodel area, that's elaborated and extended. The set of rules extends the iterative amplitude-adjusted Fourier remodel set of rules for producing surrogate and the spectral correction set of rules for simulating desk bound non-Gaussian techniques. The IPAC set of rules may be used with one-of-a-kind famous transforms, including the Fourier remodel, S-remodel, and non-stop wavelet transforms. The objectives for the simulation are the marginal chance distribution feature of the manner and the energy spectral density feature of the manner this is described primarily based totally at the variables withinside the remodel area for the followed remodel.

ISSN PRINT 2319 1775 Online 2320 7876

Research paper © 2012 IJFANS. All Rights Reserved, **UGC CARE Listed (Group -I) Journal Volume 11, Iss 10,Oct 2022**

Galerkin Method Formulation:

We are directly apply the deep Galerkin Method for solving second-order DEs without using Monte Carlo methods. In contrast to the traditional Galerkin Method, this method blends Galerkin Methods with machine learning. Rather than combining linear basis functions, the proposed method utilizes a deep neural network.

Let us consider a linear equation

$$\begin{cases} \ddot{x} + 2e\dot{x} + P(x) = f(t), \\ x(0) = 0,\ x'(0) = \dot{x}_0, \end{cases} \quad \text{.... (1)}$$

Where $x = x(t)$ and $p = p(x)$ is a coefficients.

$$P(x) = \sum_{S=0}^{M} a_s(t) x^s, x = x(t) \quad \text{........ (2)}$$

Then , we define

$$\begin{cases} \ddot{y} + 2\varepsilon\dot{y} + Q(y) = f(t) \\ y(0) = y_0 \neq 0, \text{ and } y'(0) = \dot{y}_0, \\ \qquad \text{where} \\ Q(y) = \sum_{r=0}^{M} b_r(t) y^r, y = y(t). \end{cases} \quad \text{..................... (3)}$$

The method presented is not only simpler but also more effective.

$$\varphi_j(t)\varphi_k(t)dt = 0, \quad \text{......... (4)}$$

To satisfy the differential operators and boundary conditions, we randomly sample the space points and use the gradient descent algorithm to train the neural network. This process does not require a grid [21-23]. A deep Galerkin is also capable of solving the high-dimensional DEs for this reason. The method presented is not only simpler but also more effective. Furthermore, we obtain convergence for both the loss function and the neural network.

We have

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 10,Oct 2022

$$y(t) = y_0 + x(t)$$
$$x(0) = 0, \qquad \text{....... (5)}$$
$$x^t(0) = y_0.$$

then

$$= \sum_{s=0}^{M} a_s x^s = P(x), x = x(t) \qquad \text{........ (6)}$$

Significance of Research

The technique described employs a deep neural network as opposed to a linear arrangement of basis functions. The proposed technique is meshless, and we train the neural network by randomly sampling space points and using the gradient descent algorithm to meet differential operator and boundary requirements. In addition, the convergence of the loss function and the convergence of the neural network to the precise solution in the L2 norm under specified circumstances demonstrate the solution's capacity to approach.

Results & Disussion:

A Research on the Galerkin Method for the Numerical Solution of Second-Order Linear Ordinary Differential Equations with Mixed Boundary Conditions,in explore the boundary value numerical solution provided by the finite difference technique of compare our findings to those that have previously been acquired to identify an alternative solution to the challenges to analyze standard and linear ordinary differential equations.

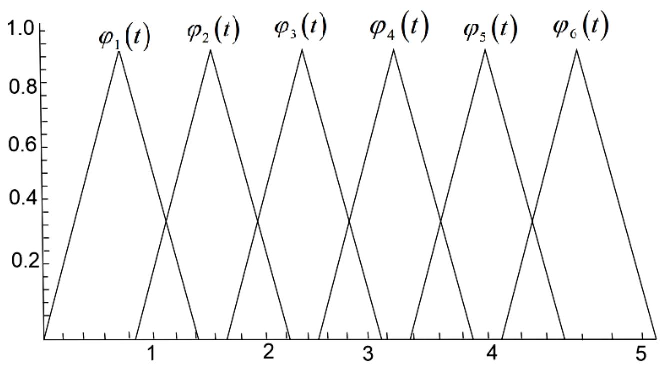

Figure 1: Randomly of Boundary Conditions

ISSN PRINT 2319 1775 Online 2320 7876
 UGC CARE Listed (Group -I) Journal Volume 11, Iss 10,Oct 2022

In develop unique iterative processes for obtaining approximate solutions to fractional differential equations, which will be depicted graphically using mathematical software.

Conclusion:

In this paper discussion of neural network by randomly sampling space points which issolving by gradient descent algorithm to meet differential operator and boundary requirements. In addition, the convergence of the loss function, and the convergence of the neural network, to the precise solution in the L2 norm under specified circumstances demonstrate the solution's capacity to approach the exact .991 Approximate Value.

References:

[1] Kulkarni, A. D. (2022). Deep Convolution Neural Networks for Image Classification.

[2] Jones, I. S., & Kording, K. P. (2021). Might a single neuron solve interesting machine learning problems through successive computations on its dendritic tree?. Neural Computation, 33(6), 1554-1571.

[3] Bengio, Y., Lecun, Y., & Hinton, G. (2021). Deep learning for AI. Communications of the ACM, 64(7), 58-65.

[4] Guglani, J., & Mishra, A. N. (2021). DNN based continuous speech recognition system of Punjabi language on Kaldi toolkit. International Journal of Speech Technology, 24(1), 41-45.

[5] Lyu, L., Zhang, Z., Chen, M., & Chen, J. (2022). MIM: A deep mixed residual method for solving high-order partial differential equations. Journal of Computational Physics, 452, 110930.

[6] Luo, H. T., Wang, L., Zhang, Y. B., Lu, G., Su, J. J., & Zhao, Y. C. (2022). Data-driven solutions and parameter discovery of the Sasa–Satsuma equation via the physics-informed neural networks method. Physica D: Nonlinear Phenomena, 440, 133-489.

[7] Wang, L., & Yan, Z. (2021). Data-driven rogue waves and parameter discovery in the defocusing nonlinear Schrödinger equation with a potential using the PINN deep learning. Physics Letters A, 404, 127-408.

[8] Haghighat, E., & Juanes, R. (2021). Sciann: A keras/tensorflow wrapper for scientific computations and physics-informed deep learning using artificial neural networks. Computer Methods in Applied Mechanics and Engineering, 373, 113552.

[9] Dufera, T. T. (2021). Deep neural network for system of ordinary differential equations: Vectorized algorithm and simulation. Machine Learning with Applications, 5, 100058.

[10] Sheng, H., & Yang, C. (2021). PFNN: A penalty-free neural network method for solving a class of second-order boundary-value problems on complex geometries. Journal of Computational Physics, 428, 110085.

[11] Schiassi, E., Furfaro, R., Leake, C., De Florio, M., Johnston, H., & Mortari, D. (2021). Extreme theory of functional connections: A fast physics-informed neural network

method for solving ordinary and partial differential equations. Neurocomputing, 457, 334-356.

[12] Sabir, Z., Nisar, K., Raja, M. A. Z., Ibrahim, A. A. B. A., Rodrigues, J. J., Al-Basyouni, K. S., ... & Rawat, D. B. (2021). Heuristic computational design of Morlet wavelet for solving the higher order singular nonlinear differential equations. Alexandria Engineering Journal, 60(6), 5935-5947.

[13] Jamili, E., & Dua, V. (2021). Parameter estimation of partial differential equations using artificial neural network. Computers & Chemical Engineering, 147, 107-221.

[14] Jiang, C., Vinuesa, R., Chen, R., Mi, J., Laima, S., & Li, H. (2021). An interpretable framework of data-driven turbulence modeling using deep neural networks. Physics of Fluids, 33(5), 055133.

[15] Li, Z., & Farimani, A. B. (2022). Graph neural network-accelerated Lagrangian fluid simulation. Computers & Graphics, 103, 201-211.

[16] Eliasof, M., Haber, E., & Treister, E. (2021). Pde-gcn: Novel architectures for graph neural networks motivated by partial differential equations. Advances in Neural Information Processing Systems, 34, 3836-3849.

[17] Zeng, S., Zhang, Z., & Zou, Q. (2022). Adaptive deep neural network methods for high-dimensional partial differential equations. Journal of Computational Physics, 463, 111-232.

[18] Kogut, P., Kohut, Y., & Parfinovych, N. (2022). Solvability Issues for Some Noncoercive and Nonmonotone Parabolic Equations Arising in the Image Denoising Problems, 30(2), 19-48.

[19] Fang, Y., & Zhang, C. (2023). Regularity for quasi-linear parabolic equations with nonhomogeneous degeneracy or singularity. Calculus of Variations and Partial Differential Equations, 62(1), 2.

[20] Singh. R, Gupta.V, Tiwari, S. K, (2016). Linear and Non Linear Principal Component Analysis, Volume 5, Issue 5, 16-21.

[21] Hong, H. P., Cui, X. Z., & Qiao, D. (2021). An algorithm to simulate nonstationary and non-Gaussian stochastic processes. Journal of Infrastructure Preservation and Resilience, 2(1), 1-15.

[22] Agarwal, A., Kakade, S. M., Lee, J. D., & Mahajan, G. (2021). On the Theory of Policy Gradient Methods: Optimality, Approximation, and Distribution Shift. J. Mach. Learn. Res., 22(98), 1-6.

[23] Singh. R, Gupta.V, Tiwari, S. K, Image Denoising using BM3D and Principal Component Analysis, Volume 6, Issue 5, 2017,1-7.

[24] A K Malik, Dipak Chakraborty, Satish Kumar. Quadratic Demand based Inventory Model with Shortages and Two Storage Capacities System. Research J. Engineering and Tech. 2017; 8(3): 213-218.

[25] Gupta, K. K., Sharma, A., Singh, P. R., Malik, A. K. Optimal ordering policy for stock-dependent demand inventory model with non-instantaneous deteriorating items. International Journal of Soft Computing and Engineering, 2013; 3(1): 279-281.

[26] Kumar, S., Chakraborty, D., Malik, A. K. A Two Warehouse Inventory Model with Stock-Dependent Demand and variable deterioration rate. International Journal of Future Revolution in Computer Science & Communication Engineering, 2017; 3(9): 20-24.

[27] Kumar, S., Malik, A. K., Sharma, A., Yadav, S. K., Singh, Y. An inventory model with linear holding cost and stock-dependent demand for non-instantaneous deteriorating items. In AIP Conference Proceedings, 2016; 1715(1): 020058.

[28] Kumar, S., Soni, R., Malik, A. K. Variable demand rate and sales revenue cost inventory model for non-instantaneous decaying items with maximum life time. International Journal of Engineering & Science Research, 2019; 9(2): 52-57.

[29] Malik, A. K. and Singh, Y. A fuzzy mixture two warehouse inventory model with linear demand. International Journal of Application or Innovation in Engineering and Management, 2013; 2(2): 180-186.

[30] Malik, A. K. and Singh, Y. An inventory model for deteriorating items with soft computing techniques and variable demand. International Journal of Soft Computing and Engineering, 2011; 1(5): 317-321.

[31] Malik, A. K., Chakraborty, D., Bansal, K. K., Kumar, S. Inventory Model with Quadratic Demand under the Two Warehouse Management System. International Journal of Engineering and Technology, 2017; 9(3): 2299-2303.

[32] Malik, A. K., Mathur, P., Kumar, S. Analysis of an inventory model with both the time dependent holding and sales revenue cost. In IOP Conference Series: Materials Science and Engineering, 2019: 594(1): 012043.

[33] Malik, A. K., Sharma, M., Tyagi, T., Kumar, S., Naik, P. J., & Kumar, P. (2022). Effect of Uncertainty in Optimal Inventory Policy for Manufacturing Products. International Journal of Intelligent Systems and Applications in Engineering, 10(1s), 102-110.

[34] Malik, A. K., Shekhar, C., Vashisth, V., Chaudhary, A. K., Singh, S. R. Sensitivity analysis of an inventory model with non-instantaneous and time-varying deteriorating Items. In AIP Conference Proceedings, 2016; 1715(1): 020059.

[35] Malik, A. K., Singh, S. R., Gupta, C. B. An inventory model for deteriorating items under FIFO dispatching policy with two warehouse and time dependent demand. Ganita Sandesh, 2008; 22(1), 47-62.

[36] Malik, A. K., Singh, Y., Gupta, S. K. A fuzzy based two warehouses inventory model for deteriorating items. International Journal of Soft Computing and Engineering, 2012; 2(2), 188-192.

[37] Malik, A. K., Vedi, P., and Kumar, S. An inventory model with time varying demand for non-instantaneous deteriorating items with maximum life time. International Journal of Applied Engineering Research, 2018; 13(9): 7162-7167.

[38] Malik, A.K. and Garg, H. An Improved Fuzzy Inventory Model Under Two Warehouses. Journal of Artificial Intelligence and Systems, 2021; 3, 115–129. https://doi.org/10.33969/AIS.2021.31008.

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 10,Oct 2022

[39] Malik, A.K. and Sharma, A. An Inventory Model for Deteriorating Items with Multi-Variate Demand and Partial Backlogging Under Inflation, International Journal of Mathematical Sciences, 2011; 10(3-4): 315-321.

[40] Malik, A.K., Singh, A., Jit, S., Garg. C.P. "Supply Chain Management: An Overview". International Journal of Logistics and Supply Chain Management, 2010; 2(2): 97-101.

[41] Satish Kumar, Yashveer Singh, A. K. Malik. An Inventory Model for both Variable Holding and Sales Revenue Cost. Asian J. Management; 2017; 8(4):1111-1114.

[42] Sharma, A., Gupta, K. K., Malik, A. K. Non-Instantaneous Deterioration Inventory Model with inflation and stock-dependent demand. International Journal of Computer Applications, 2013; 67(25): 6-9.

[43] Singh, S. R. and Malik, A. K. Effect of inflation on two warehouse production inventory systems with exponential demand and variable deterioration. International Journal of Mathematical and Applications, 2008; 2(1-2): 141-149.

[44] Singh, S. R. and Malik, A. K. Inventory system for decaying items with variable holding cost and two shops, International Journal of Mathematical Sciences, 2010; 9(3-4): 489-511.

[45] Singh, S. R. and Malik, A. K. Two warehouses model with inflation induced demand under the credit period. International Journal of Applied Mathematical Analysis and Applications, 2009; 4(1): 59-70.

[46] Singh, S. R., Malik, A. K. An Inventory Model with Stock-Dependent Demand with Two Storages Capacity for Non-Instantaneous Deteriorating Items. International Journal of Mathematical Sciences and Applications, 2011; 1(3): 1255-1259.

[47] Singh, S. R., Malik, A. K., & Gupta, S. K. Two Warehouses Inventory Model for Non-Instantaneous Deteriorating Items with Stock-Dependent Demand. International Transactions in Applied Sciences, 2011; 3(4): 911-920.

[48] Singh, Y., Arya, K., Malik, A. K. Inventory control with soft computing techniques. International Journal of Innovative Technology and Exploring Engineering, 2014; 3(8): 80-82.

[49] Singh, Y., Malik, A. K., Kumar, S., An inflation induced stock-dependent demand inventory model with permissible delay in payment. International Journal of Computer Applications, 2014; 96(25): 14-18.

[50] Tyagi, T., Kumar, S., Malik, A. K., & Vashisth, V. (2022). A novel neuro-optimization technique for inventory models in manufacturing sectors. Journal of Computational and Cognitive Engineering.

[51] Tyagi, T., Kumar, S., Naik, P. J., Kumar, P., & Malik, A. K. (2022). Analysis of Optimization Techniques in Inventory and Supply Chain Management for Manufacturing Sectors. Journal of Positive School Psychology, 6(2), 5498-5505.

[52] Vashisth, V., Tomar, A., Chandra, S., Malik, A. K. A trade credit inventory model with multivariate demand for non-instantaneous decaying products. Indian Journal of Science and Technology, 2016; 9(15): 1-6.

ISSN PRINT 2319 1775 Online 2320 7876
Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 10,Oct 2022

[53] Vashisth, V., Tomar, A., Soni, R., Malik, A. K. An inventory model for maximum life time products under the Price and Stock Dependent Demand Rate. International Journal of Computer Applications, 2015; 132(15): 32-36.

[54] Verma, P., Chaturvedi, B. K., & Malik, A. K. (2022). Comprehensive Analysis and Review of Particle Swarm Optimization Techniques and Inventory System, International Journal on Future Revolution in Computer Science & Communication Engineering, 8(3), 111-115.

[55] Yadav, S.R. and Malik, A.K. Operations Research, Oxford University Press, New Delhi, 2014.

[56] Yadav, V., Chaturvedi, B. K., & Malik, A. K. (2022). Advantages of fuzzy techniques and applications in inventory control. International Journal on Recent Trends in Life Science and Mathematics, 9(3), 09-13.