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BEST PROXIMITY POINTS ON (ψ, ϕ) CONTRACTIONS IN RMS

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Abstract

Existence of BPP of a GC(ψ, ϕ)- contractive mappings on CRMS is discussed and an example is enumerated.

Key words: RMS, BPP, p −property, Hausdroff.

1. Preliminaries and Introduction

During the last few years, FPT was one of the most developing disciplines in analysis. The development of this idea has sped up research, resulting in a massive rise in publishing [1-8].The traditional idea of MS has developed in several other areas by partially modifying the metric requirements in a vast area of works. Mathews [6-7] introduced PMS, which is among these generalisations. We see RMS described by Branciari [12] in [1, 3, 9-12].

Branciari defined an RMS and also demonstrated a Banach contraction principle analogy. The nature of these concepts and FPT for several RMS contractions has been produced in [13-18].Boyd and Wong [19] introducedCM called ϕ −contractions. In [20] a concept of weak ϕ −contractions was used andgeneralized by Alber and Guerre," A map T on a MS(M, d) is called WC if a map $\phi: [0, \infty) \to [0, \infty)$having $\phi(0) = 0$ and $\phi(t) > 0$for every$t > 0$ s.t

$d(Ta, Tb) \le d(a, b) - \phi(d(a, b))$--- (1)

for all a,$b \in M$". The abovecontractions werediscussed by many [20-23]. This type of $(\psi - \phi)$WCM has been anarea of interest in [10, 20-23].In the recent developmentin [23] we see that FPTwas got by using$(\psi - \phi)$WCM on CRMS. Here, we try to extend the result of Erhan.IM, and his companions [23] for the existence of BPP of $(\psi - \phi)$ contractions on RMS.

Definition: 1.1[12]

Let M be a set and $d: M \times M \to [0, \infty)$that satisfies the axiomsfor everya,$b \in M$ and each is distinct a ,$b \in M$ varies from c and d.

(i) $d(a, b) = 0$ iff a= b
(ii) $d(a, b) = d(b, a)$
(iii) $d(a, b) \le d(a, c) + d(c, d) + d(d, b)$

Then$d$ is called RM and (M, d) is called RMS.

Definition: 1.2

(i) A sequence $\{a_n\}$ is convergent to a limit aif and only if $d(a_n, a) \to 0$ as $n \to \infty$ (denoted by $a_n \to a$)
(ii) A sequence $\{a_n\}$ is a Cif and only if for every$\varepsilon > 0$ there is a PI N such that $d(a_n, a_m) < \varepsilon$ for all$n, m > N$.
(iii) A RMS is complete if every CS in M$converges$in M.

The modified notation of SametandLakzian [9] is vividly seenand let ψbe the set of CF. $\psi: [0, \infty) \to [0, \infty)$where$\psi(t) = 0$ iff $t = 0$NDFthat is known as ADF [23].

Theorem: 1.1[23]

Let (M, d) be a HS and CRMS and let $T: M \to M$be a self- mapthat satisfies $\psi\big(d(Ta, Tb)\big) \le \psi\big(d(a, b)\big) - \phi(d(a, b))$ for all a, $b \in M$where $\psi, \phi \in \Psi$and ψ is considered as ND and continuous.Then T has a UFP.

Definition: 1.3[2]

$A_0 = \{a \in A: d(a, b) = d(A, B)$, for b∈ B

$B_0 = \{b \in B: d(a, b) = d(A, B)$, for a∈ A

where $d(A, B) = inf\{d(a, b): a \in A, b \in B\}$

Definition: 1.4[25]

Let (C, D) be a pair of sets of MS$(M, d)$with$C_0 \neq 0$. The pair (C, D) is said to havea pp if and only if for any $c_1, c_2 \in C_0$ and $d_1, d_2 \in D_0, d(c_1, d_1) = d(C, D) = d(c_2, d_2)$

2. Main Results

Result: 2.1

Let (M, d) be a HS and CRMS and Let (C, D) be a pair of CSS of MSs.t C_0 is NE. $T: C \to D$ be a map that satisfies $T(C_0) \subset D_0$. Suppose

$\psi(d(Tc, Td)) \le \psi(M(c, d) - d(C, D)) - \phi(M(c, d) - d(C, D))$……....... (2)

for all c∈ $C, d \in D$ and $\psi, \phi \in \Psi$ where $L > 0$,and ψ is considered as ND and $M(c, d) = max\{d(c, d), d(c, Tc), d(d, Td)\}$

$$m(c, d) = min\{d(c, Tc), d(d, Td), d(c, Td), d(c, Tc)\}$$

Then T has a BPP.

Proof:

Choose $c_0 \in C$.

For$Tc_0 \in T(C_0) \subset D_0$, there exists $c_1 \in C_0$s.t $d(c_1, Tc_0) = d(C, D)$.

Similarly, regarding the assumption, $Tc_1 \in T(C_0) \subset D_0$,

We determine $c_2 \in C_0$ such that $d(c_2, Tc_1) = d(C, D)$.

We get a sequence by recursion$\{c_n\}$ in C_0 satisfying

$d(c_{n+1}, Tc_n) = d(C, D)$ for all $n \in N$..(3)

Claim: $d(c_n, c_{n+1}) \to 0$

If $c_N = c_{N+1}$, then c_N is a BPP.

By the p-property, we get

$$d(c_{n+1}, c_{n+2}) = d(Tc_n, Tc_{n+1})$$

Wesuppose that $c_n \neq c_{n+1}$ for all $n \in N$.

Since $d(c_{n+1}, Tc_n) = d(C, D)$, from (3), we have for all $n \in N$.

$\psi(d(c_{n+1}, c_{n+2})) = \psi(d(Tc_n, Tc_{n+1}))$

$\leq \psi(M(c_n, c_{n+1}) - d(C, D)) - \phi(M(c_n, c_{n+1}) - d(C, D)$ ....(4)

Where $M(c_n, ca_{n+1}) = max\,\{d(c_n, c_{n+1}), d(c_n, Tc_n), d(c_{n+1}, Tc_{n+1})$

If $M(c_{n+1}, c_{n+2}) = d(c_n, c_{n+1})$, then we get

$\psi(d(c_{n+1}, c_{n+2})) \leq \psi(d(c_n, c_{n+1}) - d(C, D)) - \phi(d(c_n, c_{n+1}) - d(C, D))$.

Where $\phi(d(c_n, c_{n+1})) = 0$ and hence $d(c_n, c_{n+1}) = 0$, contradicts our assumption.

Therefore $d(c_{n+1}, c_{n+2}) < d(c_n, c_{n+1})$ for any $n \in N$ and hence $\{d(c_n, c_{n+1})\}$ is MDS of NNRN and there is $s \geq 0$ such that $\lim_{n\to\infty} d(c_n, c_{n+1}) = s$.

From (3), for any $n \in N$, we get

$$\psi(d(c_{n+1}, c_{n+2})) \leq \psi(M(c_n, c_{n+1}) - d(C, D)) - \phi(M(c_n, c_{n+1}) - d(C, D))$$

As $n \to \infty$ in the above equation and using ψ and ϕ we get

$\psi(s) \leq \psi(s) - \phi(s)$ which implies $\phi(s) = 0$

Hence $\lim_{n\to\infty} d(c_n, c_{n+1}) = 0$..(5)

Next we show that $\{c_n\}$ is a CS.

Otherwise there is $\varepsilon > 0$, for which we can get two sequences of PI(m_k) and (n_k) s.t for all PI $m_k > n_k > k$, $d(c_{m_k}, c_{n_k}) \geq \varepsilon$ and $d(c_{m_k}, c_{n_{k-1}}) < \varepsilon$.

Now $\varepsilon \leq d(c_{m_k}, c_{n_k}) \leq d(c_{m_k}, c_{n_{k-1}}) + d(c_{n_{k-1}}, c_{n_k})$,

$$\varepsilon \leq d(c_{m_k}, c_{n_k}) < \varepsilon + d(c_{n_{k-1}}, c_{n_k})$$

As $k \to \infty$ in the above equation and using (5) we get

$\lim_{k\to\infty} d(c_{m_k}, c_{n_k}) = \varepsilon$..(6)

Again $d(c_{m_k}, c_{n_k}) \leq d(c_{m_k}, c_{m_{k+1}}) + d(c_{m_{k+1}}, c_{n_{k+1}}) + d(c_{n_{k+1}}, c_{n_k})$.

As $k \to \infty$ in the above equations and using (5) and (6) we get

$\lim_{k\to\infty} d(c_{m_{k+1}}, c_{n_{k+1}}) = \varepsilon$..(7)

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Again $d(c_{m_k}, c_{n_k}) \leq d(c_{m_k}, c_{n_{k+1}}) + d(c_{n_{k+1}}, c_{n_k})$

Letting $k \to \infty$ in the above equations and using (5) and (6) we get

$\lim_{k\to\infty} d(c_{m_k}, c_{n_{k+1}}) = \varepsilon$…………………………………………………..….(8)

$\lim_{k\to\infty} d(c_{n_k}, c_{m_{k+1}}) = \varepsilon$ ………………………………………..………………(9)

For c= c_{m_k}, d= c_{n_k} we have

$$d(c_{m_k}, Tc_{m_k}) - d(C,D) \leq d(c_{m_k}, c_{m_{k+1}}) + d(c_{m_{k+1}}, Tc_{n_k}) - d(C,D)$$

$$= d(c_{m_k}, c_{m_{k+1}})$$

Similarly$d(c_{n_k}, Tc_{n_k}) - d(C,D) = d(c_{m_k}, c_{n_{k+1}})$and

$$d(c_{m_k}, Tc_{m_k}) - d(C,D) = d(c_{n_k}, c_{m_{k+1}})$$

From (1) we get$\psi(d(c_{m_{k+1}}, c_{n_{k+1}})) = \psi(d(Tc_{m_k}, Tc_{n_k}))$

$\leq \psi(M(c_{m_{k+1}}, c_{n_{k+1}}) - d(C,D)) - \phi(M(c_{m_{k+1}}, c_{n_{k+1}}) - d(C,D)) - Lm(c_{m_{k+1}}, c_{n_{k+1}}) - d(C,D)$…………………………………….(10)

Where $M(c_{m_{k+1}}, c_{n_{k+1}}) = max\{d(c_{m_k}, c_{m_{k+1}}), d(c_{m_k}, Tc_{m_k}), d(c_{m_{k+1}}, c_{n_{k+1}})\}$

$$m(c_{m_{k+1}}, c_{n_{k+1}}) = min\{d(c_{m_k}, c_{m_{k+1}}), d(c_{m_k}, Tc_{m_k}), d(c_{m_{k+1}}, c_{n_{k+1}})\}$$

Recalling (5), (6), (7) and (8) and let$k \to \infty$ in the above equations and usingψandϕ, we get

$\psi(\varepsilon) \leq \psi(\varepsilon) - \phi(\varepsilon) + 0$……………………………………...………….(11)

Thisleads to$\phi(\varepsilon) = 0$ and hence$\varepsilon = 0$.

This contradicts our assumption.

Hence $\{c_n\}$ is a CS.

Since $\{c_n\} \subset C$and C is a CSS of the CMS(M,d), there is c^* in C s.t$c_n \to c^*$.

Taking c= $c_n$and d= $c^*$and since

$d(c_n, Tc^*) \leq d(c_n, c^*) + d(c^*, Tc_n)$ and

$d(c^*, Tc_n) \leq d(c^*, Tc^*) + d(Tc^*, Tc_n)$

We get$\psi(d(c_{n+1}, Tc^*) - d(C,D) \leq \psi d(Tc_n, Tc^*))$

$$\leq \psi(M(c_n, c^*) - d(C,D)) - \phi(M(c_n, c^*) - d(C,D)) + Lm(c_n, c^*) - d(C,D)$$

As the limit $n \to \infty$ in equations and usingψ and ϕ,we get

$$\psi(d(c^*, Tc^*) - d(C,D)) \leq \psi(d(c^*, Tc^*) - d(C,D)) - \phi(d(c^*, Tc^*) - d(C,D)) + Ld(c^*, Tc^*) - d(C,D)$$

This implies that$d(c^*, Tc^*) = d(C,D)$

Hence c^* is a BPP of T.

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To prove uniqueness

Let c and d be two BPP and assumethat $c \neq d$,

Taking c= e and d= f in (1) we get

$$\psi(d(T_e, T_f)) \leq \psi(d(e,f) - d(C,D)) - \phi(d(e,f) - d(C,D)) + Lm(e,f) - d(C,D$$

By ψ.and ϕ there is a contradiction.

Therefore e= f

Corollary: 2.1

Let (M,d) be a HS and CRMS and Let (C,D) be a pair of closed subsets of MSs.t C_0 is nonempty. Let $T: C \rightarrow D$ be a map thatsatisfies$T(C_0) \subset D_0$. Suppose

$\psi(d(Tc,Td)) \leq \psi(M(c,d) - d(C,D)) - \phi(M(c,d) - d(C,D))$.……...... (12)

for all c∊ $C, d \in D$where$\psi, \phi \in \Psi$andψ is taken as ND and $M(c,d) = max\{d(c,d), d(c,Tc), d(d,Td)\}$

$$M(c,d) = max\{d(c,d), d(c,Tc), d(d,Td)\}$$

Then T has a BPP.

Proof

Observe

$$\psi(d(Tc,Td)) \leq \psi(M(c,d) - d(C,D)) - \phi(M(c,d) - d(C,D))$$

$\leq \psi(M(c,d) - d(C,D)) - \phi(M(c,d) - d(C,D)) + Lm(c,d) - d(C,D)$ for some $L > 0$

Then by the Result 2.1, T has a BPP.

Corollary: 2.2

Let (M,d) be a HS and CRMS and Let (C,D) be a pair of subsets of MSs.tC_0 is nonempty. Let $T: C \rightarrow D$ be a map satisfy$T(C_0) \subset D_0$. Suppose

$d(Tc,Td) \leq k\ max\{d(c,d), d(c,Tc), d(d,Td)\}$.…….................................. (13)

for all c∊ $C, d \in D$for $0 \leq k < 1$.

Then T has a BPP.

Proof

Let $\psi(t) = t$ and $\phi(t) = (1-k)t$.

Byusing the Result2.2T has a BPP.

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Corollary: 2.3

Let (M,d) be a HS and CRMS and Let (C,D) be a pair of subsets of MSs.t C_0 is nonempty. Let $T\colon C \to D$ be a map satisfy$T(C_0) \subset D_0$. Suppose

$d(Tc,Td) \ \leq k[\{d(c,d)+d(c,Tc)+d(d,Td)\}-d(C,D)]+Lmin[d(c,d)+d(c,Tc)+ d(d,Td)]$……... (14)

for all c∈ $C, d \in D$ and $0 \leq k < \frac{1}{3}$and$L > 0$.

Then T has a BPP.

Proof

Clearly, $k[\{d(c,d)+d(c,Tc)+d(d,Td)\}-d(C,D)]+L\, min\{d(c,Tc),d(d,Td),d(c,Td),d(d,Tc)\}$

$$\leq 3k\, max\{d(c,d),d(c,Tc),d(d,Td)\}+L\, min\{d(c,Tc),d(d,Td),d(d,Tc)\}$$

Taking$\psi(t)=t$ and $\phi(t)=(1-3k)t$.

By the Result 2.1, T has a BPP.

Corollary: 2.4

Let (M,d) be a HS and CRMS and Let (C,D) be a pair of subsets of MSs.t C_0 is nonempty. Let $T\colon C \to D$ be a map satisfy $T(C_0) \subset D_0$. Suppose

$d(Tc,Td) \ \leq \psi(M(c,d)-d(C,D))-\phi(M(c,d) \ -d(C,D))+Lm(c,d)- d(C,D)$……... (15)

for all c∈ $C, d \in D$ where $M(c,d)=max\{d(c,d),d(c,Tc),d(d,Td)\}$

$$m(c,d)=min\{d(c,d),d(c,Tc),d(d,Td)\}$$

Then T has a BPP.

Proof: Let $\psi(t)=t$

By the Result 2.1, T has a BPP.

Example: 2.1

Let$C \cup D = M$, where $C=\left\{\frac{1}{4},\frac{1}{5},\frac{1}{6},\frac{1}{7}\right\} D=\{1,2\}$

Define the GMS M as follows:

$$d(\frac{1}{4},\frac{1}{5})=d(\frac{1}{6},\frac{1}{7})=0.05$$

$$d(\frac{1}{4},\frac{1}{7})=d(\frac{1}{5},\frac{1}{6})=0.03$$

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$$d(\frac{1}{4},\frac{1}{6}) = d(\frac{1}{5},\frac{1}{7}) = 0.08$$

$$d(\frac{1}{5},\frac{1}{5}) = d(\frac{1}{6},\frac{1}{6}) = 0$$

and $d(c,d) = |c-d| - d(C,D)$ if c, $d \in D$ (or) c$\in C, d \in D$ (or)

$$c \in D, d \in C.$$

Obviously d does not holdTon C. Indeed

$$0.08 = d(\frac{1}{4},\frac{1}{6}) \geq d(\frac{1}{4},\frac{1}{5}) + d(\frac{1}{5},\frac{1}{6}) = 0.08$$

RMS holds, and d is a RM.

Let $T: C \to D$ be defined as

$$Tc = \begin{cases} \frac{1}{7}, & if\ c \in [1,2] \\ \frac{1}{6}, & if\ c \in [\frac{1}{4},\frac{1}{5},\frac{1}{6}] \\ \frac{1}{5}, & if\ a = \frac{1}{7} \end{cases}$$

Taking$\Psi(t) = t$ and $\phi(t) = \frac{t}{7}$.

TSatisfiesthe Result 2.1 and has a BPP $d(c,d) = d(C,D)$

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