e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

DOMINATION TOPIC

SELVI. S
Research Scholar
M.Phil Mathematics
Bharath Institute Of Higher Education And Research
Mail Id : mselvarajan77@gmail.com

Dr. N. RAMYA
Head of the Department, Department Of Mathematics
Bharath Institute Of Higher Education And Research

Address for Correspondence
SELVI. S
Research Scholar
M.Phil Mathematics
Bharath Institute Of Higher Education And Research
Mail Id : mselvarajan77@gmail.com

Abstract

Graph Theory is a delightful playground for the exploration of proof techniques in discrete mathematics, and its results have applications in many areas of computing, social, and natural sciences. How can we lay cable at minimum cost to make every telephone reachable from every other? What is the fastest route from the national capital to each state capital? How can n jobs be filled by n people with maximum total utility? What is the maximum flow per unit time from source to sink in a network of pipes? How many layers does a computer chip need so that wires in the same layer don't cross? How can the season of a sports league be scheduled into the minimum number of weeks? In what order should a travelling salesman visit cities to minimum number of weeks? Can we colour the regions of every map using four colours so that neighbouring regions receive different colours? These and many other practical problems involve graph theory (D. B. West, 2002 page1).

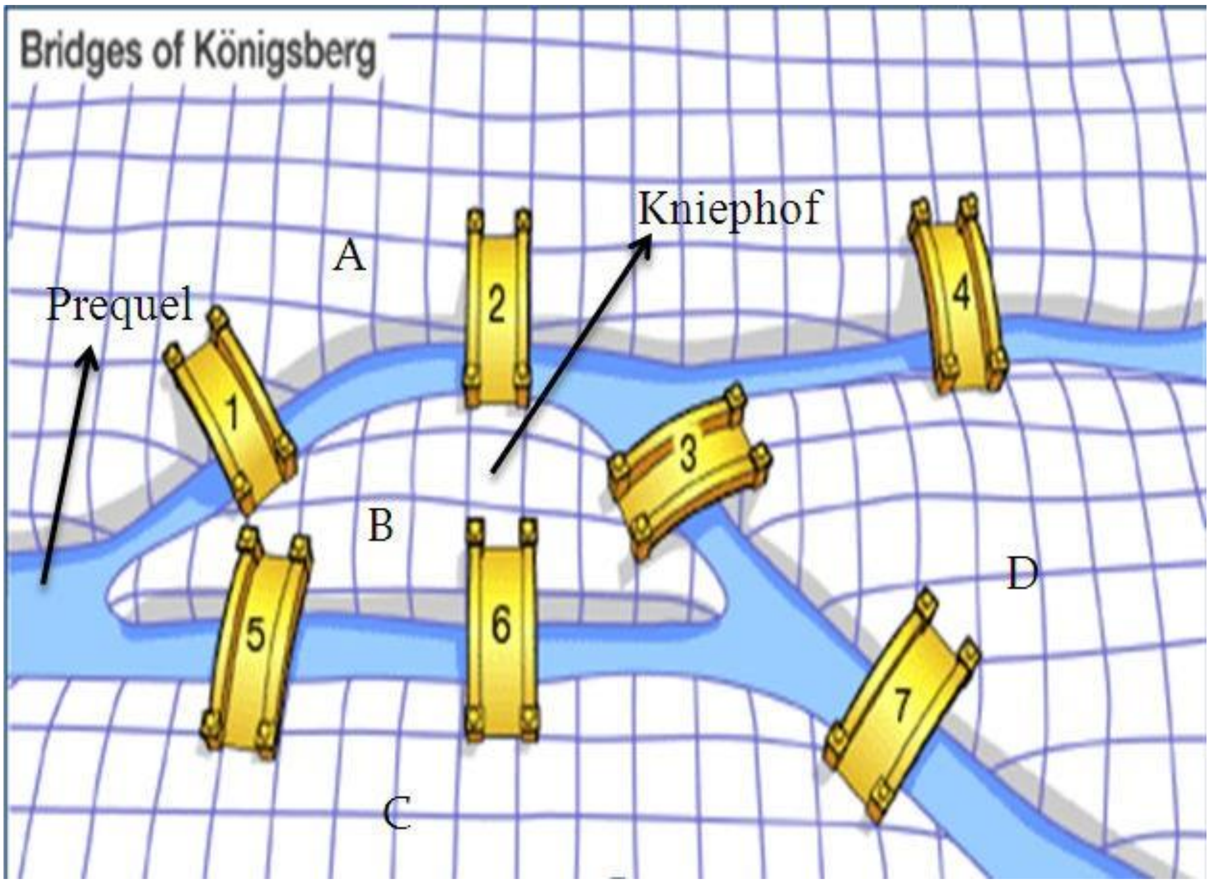
Graph Theory was born in 1936 with Euler's paper in which he solved the Konigsberg Bridge problem. The past 50 years has been a period of intense activity in graph theory in both pure and applied mathematics. Perhaps the fastest-growing area within graph theory is

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

the study of domination and related subset problems, such as independence, covering, and matching.

History of Graph theory

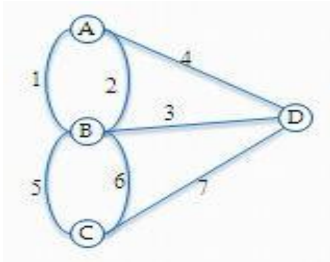
Konigsberg is a city which was the capital of East Prussia but now is known as Kaliningrad in Russia. The city is built around the River Pregel where it joins another river. An island named Kniephof is in the middle of where the two rivers join. There are seven bridges that join the different parts of the city on both sides of the rivers and the island.



People tried to find a way to walk all seven bridges without crossing a bridge twice, but no one could find a way to do it. The problem came to the attention of a Swiss mathematician named Leonhard Euler (pronounced "oiler").

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

In 1735, Euler presented the solution to the problem before the Russian Academy. He explained why crossing all seven bridges without crossing a bridge twice was impossible. While solving this problem, he developed a new mathematics field called graph theory, which later served as the basis for another mathematical field called topology



Euler simplified the bridge problem by representing each land mass as a point and each bridge as a line. He reasoned that anyone standing on land would have to have a way to get on and off. Thus each land mass would need an even number of bridges. But in Konigsberg, each land mass had an odd number of bridges. This was why all seven bridges couldnot be crossed without crossing one more than once.

The Konigsberg Bridge Problem is the same as the problem of drawing the above figure without lifting the pen from the paper and without retracing any line and coming back to the starting point.

Concept of Domination in graphs

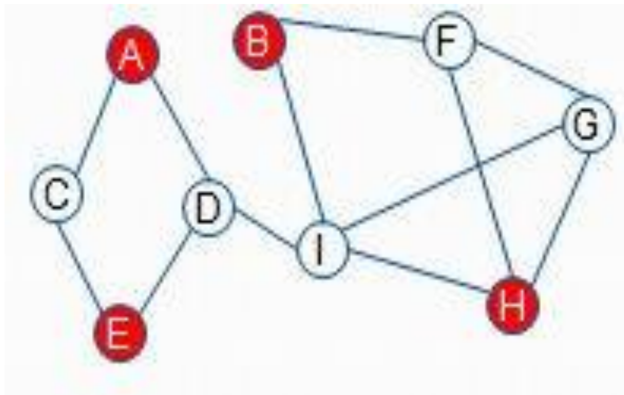
In this chapter we collect the basic definitions and theorems on domination in graphs which are needed for the subsequent chapters.

3.1 Dominating set : In graph theory, a dominating set for a graph G = (V, E) is a subset D of V such that every vertex not in D (every vertex in V- D) is joined to at least one member of D by some edge.

(i.e.) A set D of vertices in a graph G is called a dominating set of G if every vertex in V-D is adjacent to some vertex in D.

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

Ex. In the following graph G

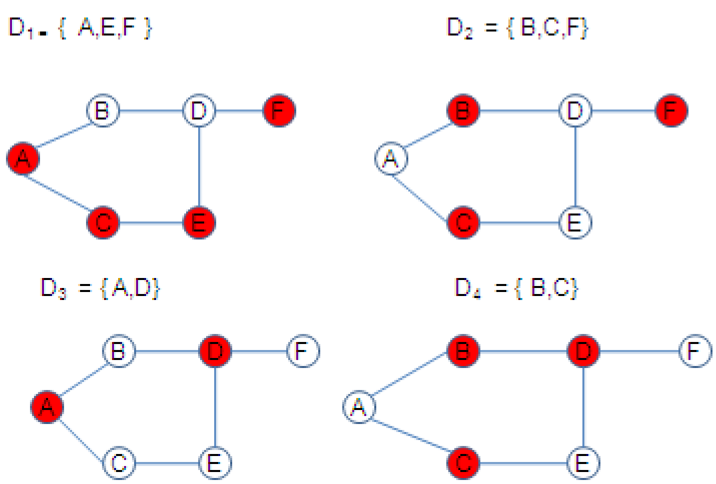


The set D = {A, B, E, H} is one of the dominating set

Minimum Dominating set:

A dominating set D is said to be Minimum Dominating set if D consist of minimum number of vertices among all dominating sets..

Ex. In the following graph G



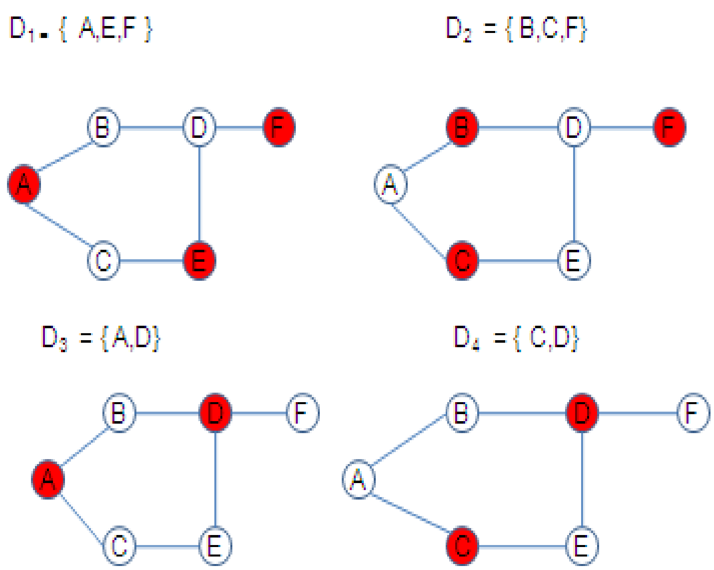
Here D_3 is the minimum dominating set

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

Domination number:

The domination number γ (G) is the number of vertices in a smallest dominating set for G. (The cardinality of minimum dominating set)

Ex. In the following graph G

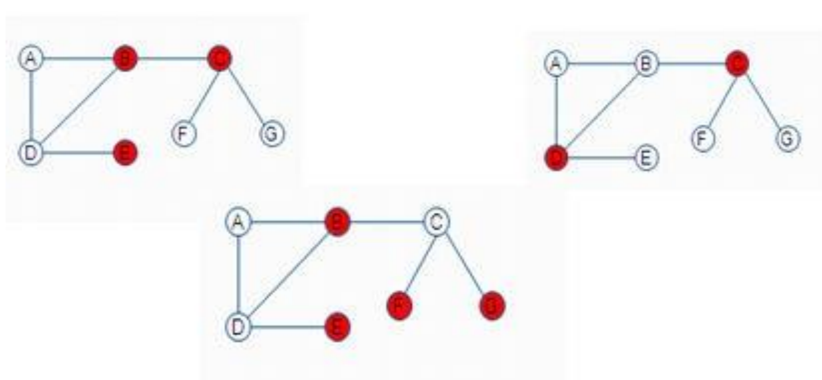


Here D_3, D_4 are the **minimum dominating set. So** γ(G) = 2

Minimal Dominating Set:

A dominating set D is called Minimal dominating set if no proper subset of D is a dominating set

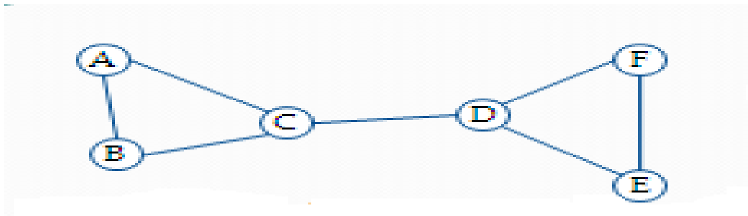
Ex.



e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022
Research Paper

The sets {B,C,E} ,{D,C} and {B,E,F,G} are Minimal dominating sets.

In the following graph



The set D_1 = {B, C, D} is a dominating set. But D_1 is not a minimal dominating set.

D_2 = {C, D} is a minimal dominating set. Also D_2 is a minimum dominating set.

A minimum dominating set is a minimal dominating set, but the converse is not always true.

Theorem 2.1: A dominating set D is a minimal dominating set if and only if for each vertex v□D, one of the following two conditions holds:

(a) v is an isolated vertex of D

(b) there exists a vertex u □ V-D such that N(u) □ D = {v}.

Theorem 2.2: Every connected graph G of order n □ 2 has a dominating set D whose complement V-D is also a dominating set.

Changing and unchanging Domination parameters

An Important Consideration in the topological design of a network is fault tolerance, that is, the ability of the network to provide service even when it contains a faulty component or components. The behavior of a network in the presence of a fault can be analyzed by determining the effect that removing an edge (link failure) or a vertex (processor failure) from its underlying graph G has on the fault- tolerance criterion. For example, a ϒ-set in G represents a minimum set of processors that can communicate directly with all other processors in the system. If it is essential for file servers to have this property and that the number of processors designated as file servers be limited, then the domination number of G is the fault-tolerance criterion. In this example, It is most important that ϒ (G) does not increase when G is modified by removing a vertex or an edge. From

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

another perspective, networks can be made fault-tolerant by providing redundant communication links (adding edges). Hence, we examine the effects on ϒ (G) when G is modified by deleting a vertex or deleting or adding an edge

Terminology:

The semi-expository paper by Carrington, Harary, and Haynes surveyed the problems of characterizing the graphs G in the following six classes. Let G-v (respectively, G-e) denote the graph formed by removing vertex v (respectively, edge e) from G. We use acronyms to denote the following classes of graphs (C represents changing; U represents unchanging; V: vertex; E: Edge; R: removal; A: addition).

(CVR) $\gamma(G - v) \neq \gamma(G)$ for all $v \in V$

(CER) $\gamma(G - e) \neq \gamma(G)$ for all $e \in E$

(CEA) $\gamma(G + e) \neq \gamma(G)$ for all $e \in E(\overline{G})$

(UVR) $\gamma(G - v) = \gamma(G)$ for all $v \in V$

(UER) $\gamma(G - e) = \gamma(G)$ for all $e \in E$

(UEA) $\gamma(G + e) = \gamma(G)$ for all $e \in E(\overline{G})$

These six problems have been approached individually in the literature with other terminology. Hence we examine them and several related problems using the above “changing and unchanging” terminology first suggested by Harary [F. Harary, Changing and unchanging invariants for graphs. Bull. Malaysian Math. Soc. 5 (1982) 73-78.

It is useful to partition the vertices of G into three sets according to how their removal affects their γ (G). Let $V = V^0 \cup V^+ \cup V^-$ for

$$V^0 = \{v \in V : \gamma(G - v) = \gamma(G)\}$$

$$V^+ = \{v \in V : \gamma(G - v) > \gamma(G)\}$$

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

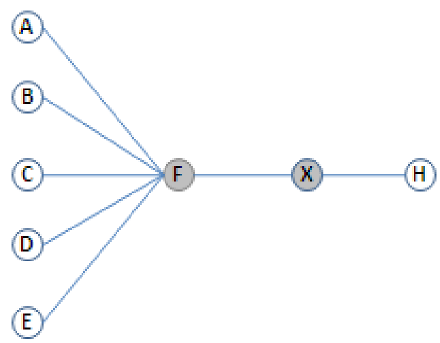
$$V^{-} = \{v \in V : \gamma(G - v) < \gamma(G)\}$$

Similarly, the edge set can be partitioned into

$$E^{0} = \{uv \in E: \gamma(G - uv) = \gamma(G)\}$$

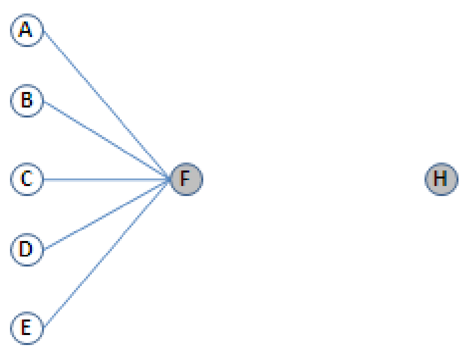
$$E^{+} = \{uv \in E: \gamma(G - uv) > \gamma(G)\}$$

For Example, the graphs in the following figure G:



$$\gamma(G) = 2$$

The graph G_1 : G-{X}



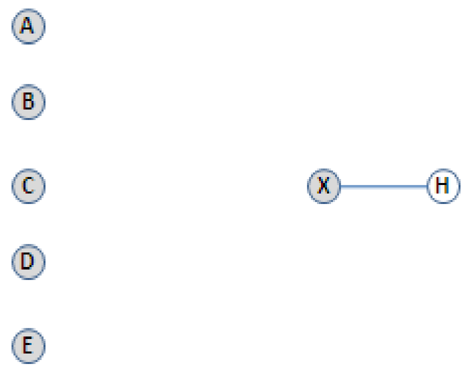
e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

γ (G-{X}) = 2

V^0 = {X □ V: γ (G – {X} = γ (G)}

(i.e.) V^0 = {A, B, C, D, E, X} ---------------------------------- □1

The graph $G_{2:}$ G-{F}



γ (G-{F}) = 6

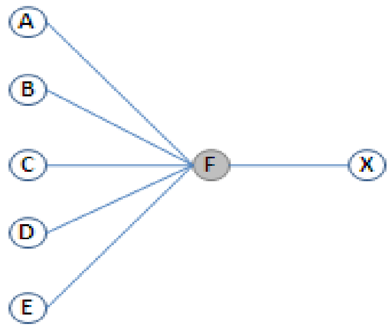
γ (G-{F}) > γ(G)

V^+ = {F □ V : γ (G – {F} > γ (G)}

(i.e.) V^+ = {F}.. □ 2

The graph G_3: G-{H}

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022



γ (G-{H}) = 1

γ (G-{H}) < γ(G)

$V^- = \{ H \square V : \gamma (G - \{H\} < \gamma (G)\}$

i.e. $V^- = \{F\}$..□3

From equation 1, 2 and 3

$V = V^0 \square V^+ \square V^-$

Vertex removal:

- The removal of vertex v from a graph G results a graph G-v such that

γ (G-v) > γ(G)

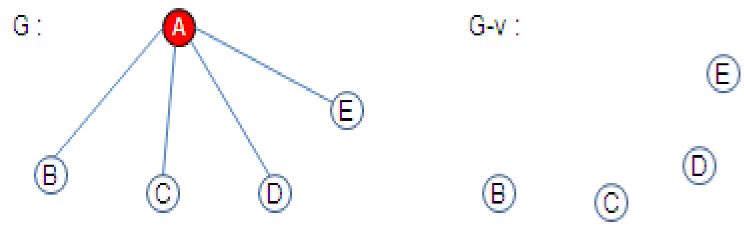
γ (G-v) < γ(G)

γ (G-v) = γ(G)

- The removal of vertex from G can increase γ(G) by more than one

Ex. For the graph $K_{1,4}$

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

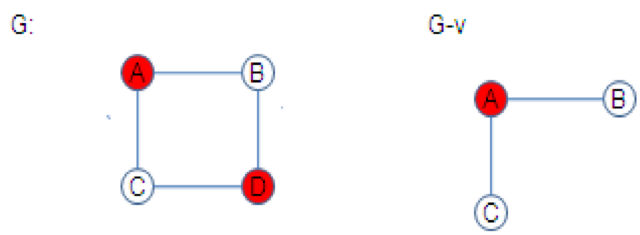


Here γ (G) = 1 γ (G-A) = 4

γ(G-A) > γ(G)

- But the removal of vertex from G can decrease γ(G) by at most one

Ex. For the cyclic graph C_4



Here γ(G) = 2 γ(G- v) = 1

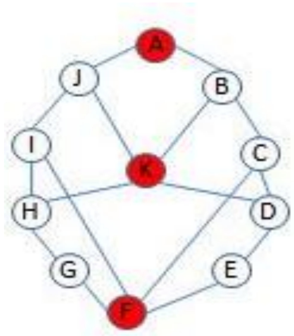
γ(G-v) < γ(G)

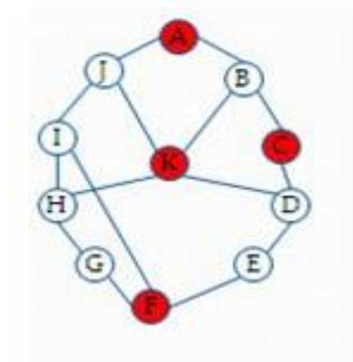
Edge removal:

- The removal of an edge from a graph G can increase by the domination number by at most one and cannot decrease the domination number. (i.e.) γ(G-e) = γ(G) + 1

Ex. G:

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022





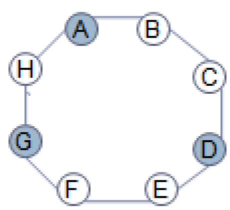
$\gamma(G) = 3$ $\gamma(G\text{-}e) = 4$

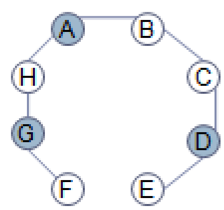
$$\gamma(G\text{-}e) = \gamma(G) + 1$$

❖ The domination number is unchanged when any single edge is removed.

$$\gamma(G\text{-}e) = \gamma(G)$$

Ex. For the cyclic graph C_8





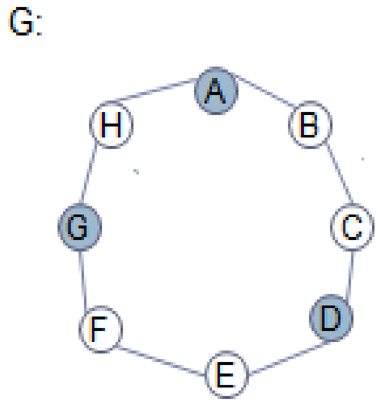
Here $\gamma(G) = 3$ $\gamma(G\text{-} e) = 3$

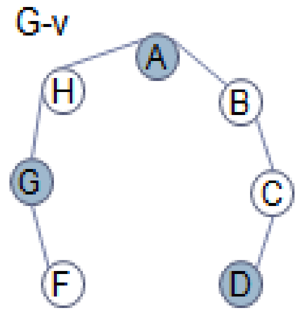
$$\gamma(G\text{-}e) = \gamma(G)$$

Vertex removal: Unchanging Domination

Ex. For the cyclic graph C_8

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022





Here $\gamma(G) = 3$ $\gamma(G-v) = 3$

$\gamma(G-v) = \gamma(G)$

4.2 Vertex removal: Changing Domination:

The vertices in V^+ were characterized by Bauer, Harary, Nieminen and Suffel [1]

Theorem 4.1[1]: A vertex $v \in V^+$ if and only if

(i) . v is not an isolated vertex and is in every γ-set of G,
and (ii). no subset $S \subseteq V - N(v)$ with cardinality $\gamma(G)$ dominates G-v.

The vertices in V^- were characterized by Sampathkumar and Neeralagi.

Theorem 4.2 [2]: A vertex $v \in V^-$ if and only if pn [v, D] = {v} for some γ-set D containing v.

Carrington et al. determined the properties of V^+ and V^- and showed that for any graph G in changing vertex removal, $\gamma(G - v) < \gamma(G)$ for all $v \in V$, that is , $V = V^-$ and $V^+ = \emptyset$.

Theorem 4.3 [3]: For any graph G,

(a) If v □ V^{+}, then for every γ-set D of G, v □ D and pn[v,D] contains at least two non adjacentvertices,

(b) if x □ V^{+} and y □ V^{-}, then x and y are not adjacent,

(c) $|V^0| \geq 2|V^+|$,

(d) $\gamma(G) \neq \gamma(G - v)$ for all v □ V if and only if $V = V^{-}$, and

(e) if v □ V^{-} and v is not an isolated vertex in G , then there exists a γ-set D of G such that v not in D

Brigham, Chinn and Dutton determined a sufficient condition to imply that $\gamma(G - v) = \gamma(G)$.They established the following theorem.

Theorem 4.4 [4]: If a graph G has a non isolated vertex v such that the subgraph induced by N (v) is complete, then $\gamma(G - v) = \gamma(G)$.

Theorem 4.5[22]: If a graph G□CVR and γ (G) □ 2, then diam (G) ≤ 2(γ (G) – 1).

Bauer et al. [1] studied a problem of determining the minimum number of vertices whose removal changes γ(G). Let □$^{+}$ denote the minimum number of vertices whose removal increases the domination number and □$^{-}$ denote the minimum number of vertices whose removal decreases the domination number They obtained the following results.

Theorem 4.6[1]: For any tree T, □$^{+}$ (T) = 2 if and only if there are vertices u and v such that

(1) every γ-set contains either u or v.

(2) v is in every γ-set for T-u and u is in every γ-set for T-v.

They also established the following results

Theorem 4.7[1]: For any graph

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

(a) □ $^{-}$(G) ≤ γ(G) + 1.

(b) min { □$^{+}$(G), □ $^{-}$(G)} ≤ □(G) + 1.

(c) If G has an end vertex, then □$^{+}$(G ≥ 2 implies □ $^{-}$(G) ≤ 2.

(d) For n ≥ 7, □ $^{+}$(P_n) + □ $^{-}$(P_n) = 4.

(e) For n ≥ 8, □ $^{+}$(C_n) + □ $^{-}$(C_n) = 6.

Bauer, Harary, Nieminen and Suffel showed that V^0 is never empty for a tree. They proved the following theorem.

Conclusion: Relationships among Classes:

There are many interesting relationships among the six classes of changing and unchanging graphs. For example, the characterization of the graphs in UEA relates them to thegraphs in CVR.

Observation 4.76:

(a) A graph G□ UVR if and only if $V = V^0$.

(b) If a graph G□ UER, then V = V^0 U $V^{□}$ U V^{+-}

(c) A graph G□ UEA if and only if V = V^0 U V^{+-} (either V^0 or V^+ may be empty).

(d) A graph G□ CVR if and only if V = $V^{□}$

(e) If a graph G□ UVR, then G □ UEA.

(f) If a graph G □ CER □ UVR if and only if G is mK_2 m □2.

(g) A graph G □ (CER □ UEA) – UVR if and only if G is a galaxy with no isolated vertices and at least one star with two or more end vertices.

(h) A graph G □ CER □(UEA □ CEA) if and only if G is a galaxy with at least

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

oneisolated vertex and at least two edges.

(i) A graph G □ CER □ CEA if and only if G has n □ 3 vertices and exactly oneedge.

(j) If a graph G □ CVR, then G □ UER.

REFERENCES

1. A Finbow, B.L.Hartnell and R. Nowakowski, Well domination in Graph, Ars Combin. 25A (1988) 5 – 10.
2. A. M. Barcalkin and L. F. German, the external stability number of the cartesian product of graphs, Bul. Akad. Stiince RSS Moldoven, No 1, 94 (1979) 5-8.
3. A.Yeo, Improved bound on the total domination in graphs with minimum degree four, manuscript (2006).
4. Alber, Jochen; Fellows, Michael R; Niedermeier, Rolf (2004), "Polynomial-time data reduction for dominating set", Journal of the ACM 51 (3): 363–384, doi:10.1145/990308.990309.
5. Allan, Robert B.; Laskar, Renu (1978), "On domination and independent domination numbers of a graph", Discrete Mathematics 23 (2): 73–76, doi: 10.1016/0012-365X(78)90105-X .
6. B. Bollobas and E. J. Cockayne, Graph theoretic parameters concerningdomination, independence and irredundance, J. Graph Theory 3 (1979) 241-250.
7. B. L. Hartnell and D. F. Rall, Bounds on the bondage number of a graph, Discrete math, 128 (1994), 173-177.
8. B. L. Hartnell, D. F. Rall, A Characterization of trees in which no edge is essential to the domination number, Arc Combin. 33 (1992) 65-76.

9. Bermond, J.C. and Peyrat, C., De Bruijn and Kautz networks: A competitor for the hypercube. In: Andre, F., Verjus, J.P. (Eds.), Hypercube and Distributed Computers, Elsevier Science, BV, Amsterdam. pp. 278-293.
10. Bollobas.B, Graph theory: An Introductory Course Springer 1979.
11. C. Bazgan: Schemas d' Approximation et Complexities Parameters. Rapport de DEA, University Paris Sud, 1995.
12. C. Berge Graphs, North-Holland 1985.
13. C. Berge, Theory of Graphs and its Applications, Methuen, London, (1962).
14. ChartChartrand.G, Introductory Graph theory, Dover 1985.
15. Cockayne, E.J., Dawes, R.M. and Hedetniemi, S.T., Total domination in graphs. Networks.v10. 211-219.
16. D. Archdeacon, J. Ellis-Monaghan, D. Fischer, D. Froncek, P.C.B. Lam, S. Seager,B. Wei, and R. Yuster, Some remarks on domination. J. Graph Theory 46 (2004),207-210.
17. D. Bauer, F. Harary, J. Nieminen and C.L. Suffel, Domination alteration sets in Graphs, Discrete Math.47(1983) 153 - 161
18. D.P.Sumner, and P.Blitch, Domination critical graphs, J. Combin. Theory, Ser. B 34 (1983) 65-76.
19. D.P.Sumner, Critical concepts in domination, Discrete Math. 86 (1990) 33-46
20. Diestel.R, Graph Theory, Springer-Verlag 1997.
21. E. J. Cockayne, S. T. Hedetniemei and D. J. Miller, Properties of hereditary hypergraphs and middle graphs, Canad. Math. Bull., 21 (1978) 461-468.
22. E. J. Cockayne, R. M. Dawes, and S. T. Hedetniemi, Total domination in graphs. Networks10 (1980), 211-219.

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

23. E. Sampathkumar and H. B. Walikar, The connected domination number of a graph, J. Math.Phys.Sci., 13 (1979) 607-613.

24. E. Sampathkumar and L.Pushpa Latha, strong (weak) domination and domination balance in graph, Discrete math, 161 (1996), 235-242.

25. E. Sampathkumar and P.S. Neeralagi, Domination neighborhood critical, fixed, free and totally free points, Sankhya (special Volume) 54 (1992) 403-407.

26. F. Harary and M. Livingston, Characterization of trees with equal dominationand independent domination number. Congr. Numer, 55 (1986) 121-150.