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Research Paper

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A VIEW ON PAIR SUM LABELLING OF UNION OF GRAPHS

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Abstract

A graph with a pair sum labeling defined on it is called a pair sum graph. Obtained from the path Pm by appending n new pendent edges at an end vertex of Pm, the path Pn by appending an edges to a vertex of the path Pm adjacent to an end point

Let Gbe a (p, q) graph. A Injective map g :V(G)→ {±1,±2…±p} is called a pair sum labeling if the induced edge function g_e:E(G)→Z-{0} defined by g_e(uv)=g(u)+g(v) is one-one and g_e(E(G)) is either of the form $\{\pm k_1, \pm k_2, \ldots, \pm k_{q/2}\}$ or $\{\pm k_1, \pm k_2, \ldots, \pm k_{q-1/2}\} \cup \{k_{q+1/2}\}$ according as q is even or odd.

Keywords:

Map, labeling, edges, graph, elements, sets, definition, etc.,

Introduction

Infinite graph

A graph is said to be infinite if it has either or both infinite number of points and lines.

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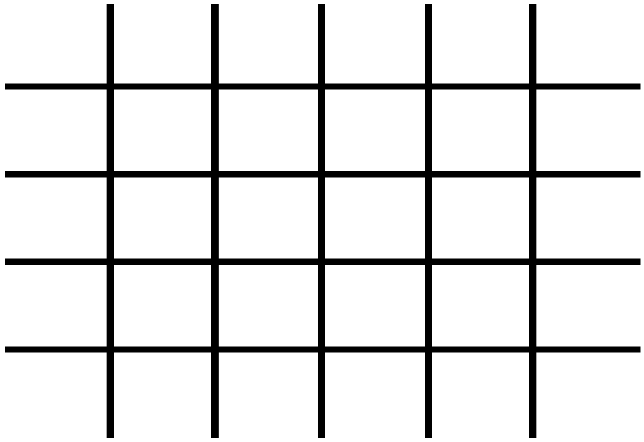
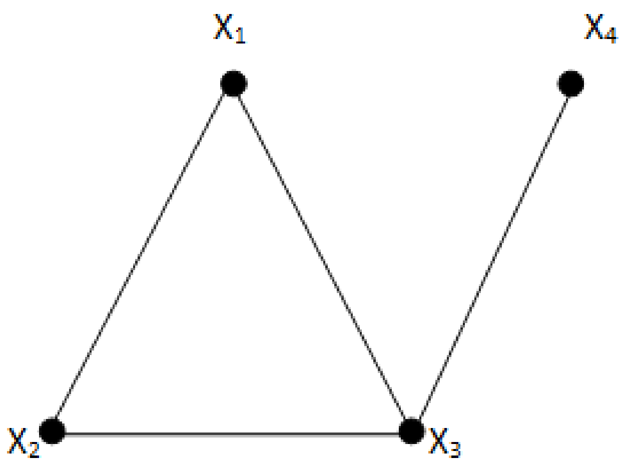


Figure

Simple graph

A simple graph G consists of a non- empty finite set V(G) of elements called vertices and a finite set E(G) of distinct un ordered pairs of distinct elements of V(G) called edges.



Figure

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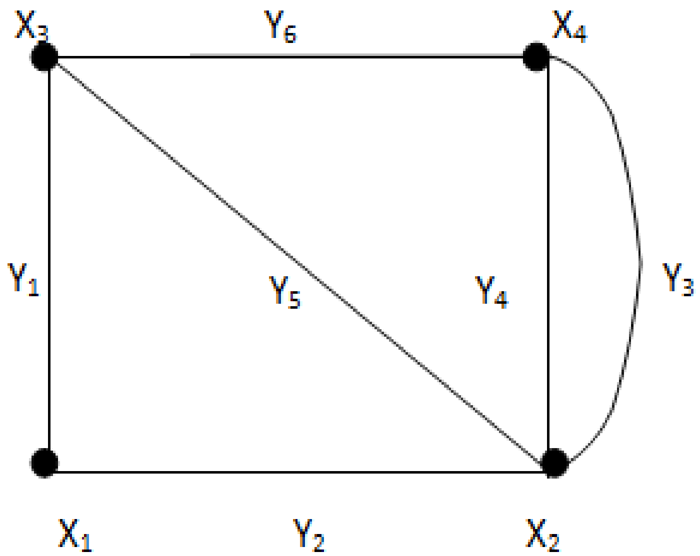
Multiple graph

A multi graph is a graph with multiple edges between the same vertices.
(i-e. either self-loop or parallel edge or both)



Figure

Definition: The union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$

Definition: If P_n denotes a path on n vertices, the graph $L_n = P_2 \times P_n$ is called a ladder .

Definition: The graph $C_n \hat{O} K_{1,m}$ is obtained from C_n and $K_{1,m}$ by identifying any vertex of C_n and central vertex of $K_{1,m}$.

Theorem

$K_{1,m} \cup K_{1,n}$ is a pair sum graph.

Proof

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Let $x, x_1, x_2, \ldots x_m$ be the vertices of $K_{1,m}$ and

$E(K_{1,m}) = \{xx_j : 1 \leq j \leq m\}$.

Let $y, y_1, y_2, \ldots, y_n$ be the vertices of $K_{1,n}$ and

$E(K_{1,n}) = \{yy_j : 1 \leq j \leq m\}$.

when n = m.

consider $g(x) = 1$

$g(x_j) = j+1 \quad 1 \leq j \leq n$

$g(y) = -1$

$g(y_j) = -(j+1) \quad 1 \leq j \leq n$

when $n > m$

consider $g(x) = 1$

$g(x_j) = j+1 \quad 1 \leq j \leq m$

$g(y) = -1$

$g(y_j) = -(j+1) \quad 1 \leq j \leq m$

$g(y_{m+2j-1}) = -(m+1+j) \quad 1 \leq j \leq \frac{n-m}{2}$ if n-m is even or

$1 \leq j \leq \frac{n-m-1}{2}$ if n-m is odd

$g(y_{m+2j}) = m+j+3 \quad 1 \leq j \leq \frac{n-m}{2}$ if n-m is even or

$1 \leq j \leq \frac{n-m-1}{2}$ if n-m is odd

Then g is a pair sum labeling.

Theorem

$P_n \cup K_{1,m}$ is a pair sum graph.

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Proof

Let $x_1,x_2,\ldots x_n$ be the path P_n. Let $V(K_{1,m})=\{y,y_j:1\leq j\leq m\}$ and

$E(K_{1,m})=\{yy_j:1\leq j\leq m\}$.

When $n = m$.

Consider $g(x)=1, \quad 1\leq j\leq n,$

$g(y)=-1,$

$g(y_j)=-2j, \quad 1\leq j\leq n,$

when $m>n$.

consider $g(x_j)=j, \quad 1\leq j\leq n,$

$g(y)=-1,$

$g(y_j)=-2j, \quad 1\leq j\leq n-1$

$g(y_{m+2j\_1})=2n+j, \quad 1\leq j\leq \frac{m-n+1}{2}$ if m-n is odd or

$1\leq j\leq \frac{m-n}{2}$ if m-n is even,

$g(y_{n+2j-2}) = -(2n+j-2) \quad 1\leq j\leq \frac{m-n+1}{2}$ if m-n is odd or

$1\leq j\leq \frac{m-n}{2}+1$ if m-n is even

Then g is a pair sum labeling.

Theorem

If $n=m$, then $C_n \cup C_m$ is a pair sum graph.

Proof

Let $x_1x_2,\ldots x_mx_1$ be the first copy of the cycle in $C_m\cup C_m$ and $y_1y_2\ldots y_my_1$ be the second copy of the cycle in $C_m\cup C_m$.

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When n = m= 4k.

Consider

$g(x_j)=j, \quad 1 \le j \le 2k-1$

$g(x_{2k})=2k+1,$

$g(x_{2k+j})=-j, \quad 1 \le j \le 2k-1,$

$g(x_m)= -2k-1,$

$g(y_j)=2k+2j, \quad 1 \le j \le 2k,$

$g(y_{2k+j})=-2k-2j,$ $g(y_{2k+j})=-2k-2j,$

$1 \le j \le 2k.$

When n = m = 4k+2

Consider

$g(x_j) = j, \quad 1 \le j \le 2k+1$

$g(x_{2k+1+j}) = -j \quad 1 \le j \le 2k+1$

$g(y_j) = 2k+2j, \quad 1 \le j \le 2k+1$

$g(y_{2k+1+j}) = -2k-2j, \quad 1 \le j \le 2k+1$

when n=m=2k+1.

$g(x_j) = -j$ and $g(y_j) = j$ we have a pair sum labeling.

$j \le 2$

Theorem

If $m \le 4$,then nK_m is a pair sum graph.

Proof

Obviously m=1,the result is true.

Case 1: m=2.

Assign the label j and j+1 to the vertices of j^{th} copy of K_2 for all odd j. For even values of j,label the vertices of the j^{th} copy of K_2 by $-j+1$ and $-j$.

Case 2: m=3.

Subcase1 m is even.

Label the vertices of first n/2 copies by 3j − 2, 3j − 1, 3j(1 ≤ j ≤ n/2). Remaining n/2 copies are labeled by -3j + 2, -3j + 1, -3j.

Subcase 2 n is odd.

Label the vertices of first (n − 1) copies as in Subcase (a). In the last copy label the vertices by $\frac{3(n-1)}{2}+1$, $\frac{-3(n-1)}{2}-2$, $\frac{3(m-1)}{2}+3$ respectively.

Case 3 m = 4

Subcase1: n is even

Label the vertices of first $\frac{n}{2}$ copies by 4j − 3, 4j − 2, 4j − 1, 4j (1 ≤ j ≤n/2). Remaining $\frac{n}{2}$ Copies are labeled by −4j+3,−4j+2,−4j+1,−4j.

Subcase2: n is odd.

Label the vertices of first (n−1) copies as in Sub case(a). In the last copy label the vertices by −2n,2n+1,2n+2 and −2n−3 respectively.

Theorem

Ladder L_m admids pair sum labeling

Proof

Let $V(L_m)=\{x_j,y_j:1\le j\le m\}$ and

$E(L_m)=\{x_jy_j:1\le j\le m\}\cup\{x_jx_{j+1},y_jy_{j+1}:1\le j\le m-1\}$.

When m is odd.

Let m=2n+1.consider $g:V(L_m)\to\{\pm1,\pm2,...,\pm(4n+2)\}$ by

$$g(x_j)=-4(n+1)+2j,\qquad 1\le j\le n,$$

$g(x_{n+1})=-(2n+1)$,

$g(x_{n+1+j})=2n+2j+2,\quad 1\le j\le n$

$g(y_j)=-4n-3+2j,\qquad 1\le j\le n$,

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$g(y_{n+1})=2n+2$

$g(y_{n+1+j})=2n+2j+1, \quad 1 \leq j \leq n.$

When m is even

Let m=2n. consider $g:V(L_m)\rightarrow\{\pm1,\pm2,...,\pm(4n+2)\}$by

$g(x_{n+1-j})=-2j, \quad 1 \leq j \leq n,$

$g(x_{n+j})=2j-1, \quad 1 \leq j \leq n,$

$g(x_{n+j})=2j, \quad 1 \leq j \leq n,$

$g(x_{n+1-j})=-(2j-1), 1 \leq j \leq n.$

Then L_m admits a pair sum labeling.

Theorem

The quadrilateral snake Q_m is a pair sum graph if m is odd.

Proof

Let $V(Q_m)=\{x_i,y_j,z_j:1 \leq i \leq m+1, 1 \leq j \leq m\}$

$E(Q_m)=\{x_iy_i,x_iz_i,x_ix_i+1,x_i+1z_i:1 \leq i \leq m\}.$

Let m=2n+1. Define $g:V(G)\rightarrow\{\pm1,\pm2,...,\pm(6n+4)\}$by

$g(x_i)=-3m+3j-4, \quad 1 \leq j \leq n+1$

$g(x_{n+j})=3m-3j+4, 1 \leq j \leq n+1$,$g(y_j)=-3m+3i-3, \quad 1 \leq j \leq n+1$

$g(y_{n+1+j})=3m-3j+3, \quad 1 \leq j \leq n$

$g(z_j)=-3m+3j-2, \quad 1 \leq j \leq n$

$g(z_{n+1})=3,$

$g(z_{n+j+1})=3m-3j+2, \quad 1 \leq j \leq n$

Then Q_m admids a pair sumlabeling.

Illustration of theorem 4.6 is shown in the following figure 4.1

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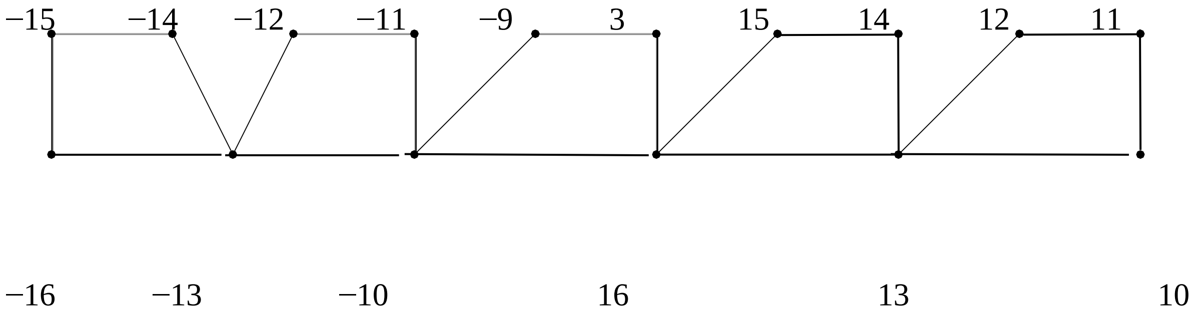

Figure

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