  

# On Rainbow Colouring Of Certain Classes Of Graphs

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**Abstract:** An edge colored graph G is rainbow edge-connected if any two vertices are connected by a path whose edges have distinct colors. We exhibit the rainbow coloring of middle graph of path Pn, Total graph of path Pn, Moser graph and square graph of comb by unique way.

**Keywords:** Rainbow coloring, Rainbow connection number, middle graph Total graph Moser graph, comb graph

## Introduction:

Let the rainbow connection of a connection graph G denoted by rc(G), be the smallest number of colors, that are needed in order to make rainbow edge-connected.. Clearly, if a graph is rainbow edge connected then it is also connected[1]. Conversely, any connected graph has a trivial edge coloring that makes it rainbow edge-connected just color each edge with a distinct color. Thus the following natural graph parameter was defined by Chartrand et al.[2]

**Preliminaries:**

**Definition1**

A path is a rainbow, if no two edges of it are colored the same. An edge-coloring graph G is rainbow connected if any two vertices are connected by a rainbow path.

**Definition 2**

The square graph $G^2$ of an undirected graph G is another graph that has the same set of vertices but in which two vertices are adjacent when their distance in G is at most 2.

The structure of square graph of comb is given below [3].

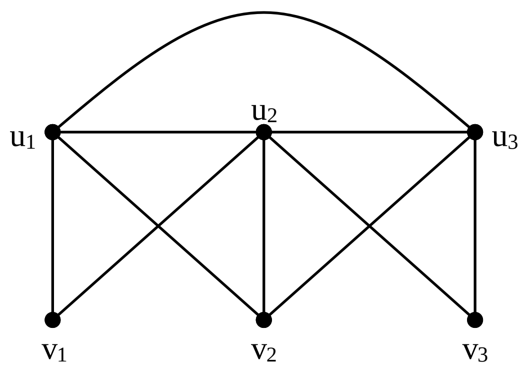


**Figure 1: Square graph of comb**

**Definition 3:**

The Moser graph which is also called Moser spindle is an undirected graph with 7 vertices and 11 edges [4].

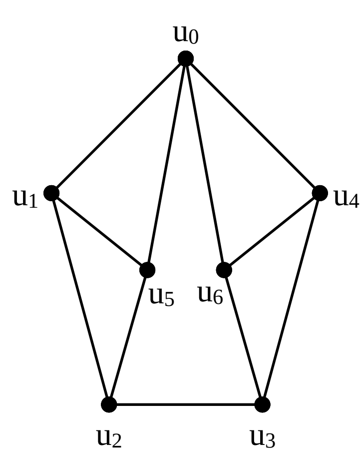


**Figure 2: Moser-spindle graph**

  

**Definition 4:**

The Middle graph M(G) of a graph G is the graph whose vertex set is V(G) ∪ E(G) and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it [5].

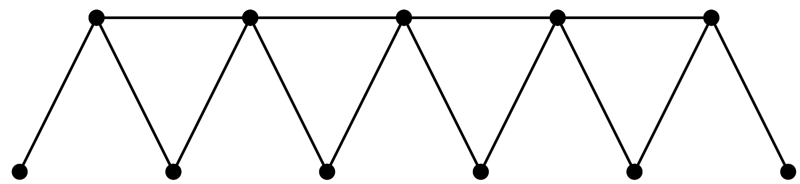

**Figure 3: Middle graph of path $P_5$**

**Definition 5:**

The total graph T(G) of a graph G is the graph whose vertex set is V(G) ∪ E(G) and two vertices are adjacent whenever they are either adjacent or incident in G [5].

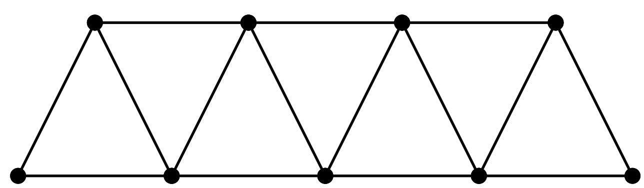

**Figure 4: Total graph of path graph**

**Main Results:**

**Theorem 1:**

Middle graph of path $P_n$ admits rainbow coloring, and whose rainbow connection number $\gamma_c(G) = n+1$ where 'n' represents number of vertices in the path.

**Proof:**

Let G = M($P_n$) and let $u_1$, $u_2$ … $u_n$ be the path vertices of G. $v_1$ $v_2$ … $v_{n+1}$ be the pendant vertices which are lower vertices of G and edge set E(G) = $\{u_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{(u_1 v_1)\} \cup \{(u_i v_{i+1} / 1 \leq i \leq n\} \cup \{u_{i+1} v_{i+1} / 1 \leq i \leq n\}$

Let $f^*$ : E(G) → {1, 2, … n} is defined as follows:

(i) $f(u_i u_{i+1}) = i+1$, i = 1, 2, … n–1

(ii) $f(u_1 v_1) = 1$

(iii) $f(u_i v_{i+1}) = i+1$, i = 1, 2, … n–1

(iv) $f(u_{i+1} u_{i+1}) = i+1$, i = 1, 2, … n–1

**Example 1:**

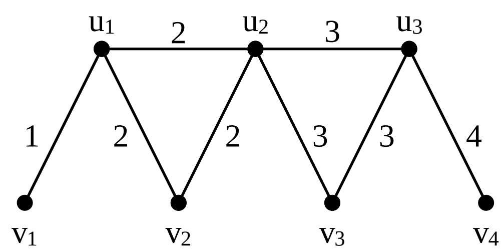


**Figure 5: Middle graph of path $P_3$**

**Example 2:**

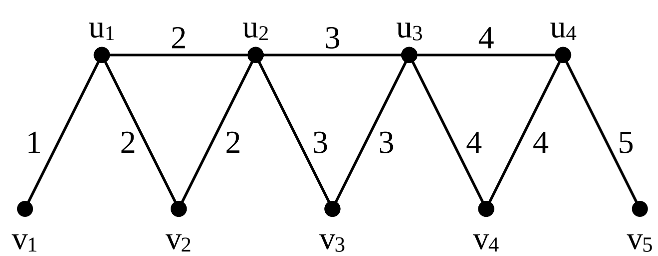


**Figure 6: Middle graph of Path $P_4$**

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**Theorem 2:**

Total graph of path $P_n$ holds rainbow coloring, and rainbow connection number is n.

**Proof:**

Let $G = T(P_n)$ and let $u_1, u_2 \dots u_n$ be the upper vertices in the path and $v_1, v_2 \dots v_{n+1}$ be the lower most vertices in the path and edge set $E(G) = \{(u_i\, u_{i+1}) \cup (u_i\, v_i) \cup (u_i\, v_{i+1}) \cup (v_i\, v_{i+1}) / 1 \leq i \leq n\}$

Let $f^* : E(G) \rightarrow \{1, 2, \dots n\}$ is defined as follows:

(i) $f(u_i\, u_{i+1}) = i+1$, for $i = 1, 2, \dots n-1$
(ii) $f(u_i\, v_i) = i$, for $i = 1, 2, \dots n-1$
(iii) $f(u_i\, v_{i+1}) = i$, for $i = 1, 2, \dots n$
(iv) $f(v_i\, v_{i+1}) = i$, for $i = 1, 2, \dots n$

**Example 1:**

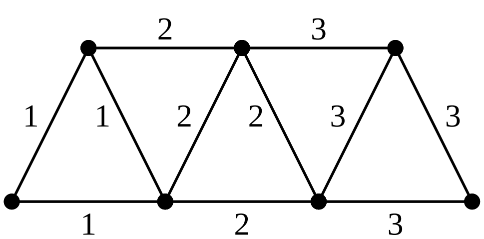


**Figure 7: Total graph $T(P_4)$**

**Example 2:**

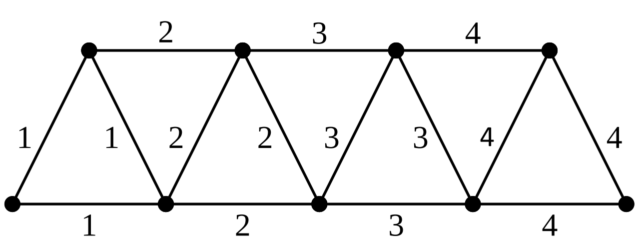


**Figure 8: Total graph $T(P_5)$**

 

**Theorem 3:**

The square graph of comb admits rainbow coloring and whose rainbow connection number is n–1 wherein 'n' represents number of vertices in the path.

**Proof:**

Let G be the square graph of comb and is denoted by $(P_n \Theta K_1)^2$ with vertex set $V = \{x_i\ y_i / 1 \leq i \leq n\}$ and edge set $E = E_1 \cup E_2 \cup E_3$ where

$E_1 = \{x_i\ x_{i+1},\ x_i\ y_{i+1}\}$, i = 1, 2, … n–1

$E_2 = \{x_i\ x_{i+2},\ y_i\ x_{i+1}\}$

$E_3 = \{x_i\ y_i\}$

Let $f^* : E(G) \rightarrow \{1, 2, \dots n–1\}$ is defined as follows:

(i) $f(x_i\ x_{i+1}) = i$, for i = 1, 2, … n
(ii) $f(x_i\ y_{i+1}) = i$, for i = 1, 2, … n
(iii) $f(x_i\ x_{i+2}) = i+1$, for i = 1, 2, … n
(iv) $f(y_i\ x_{i+1}) = i$, for i = 1, 2, … n
(v) $f(x_1\ y_1) = 2$
(vi) $f(x_i\ y_i) = i–1$, for i = 2, 3, 4, … n

**Illustration 1:**

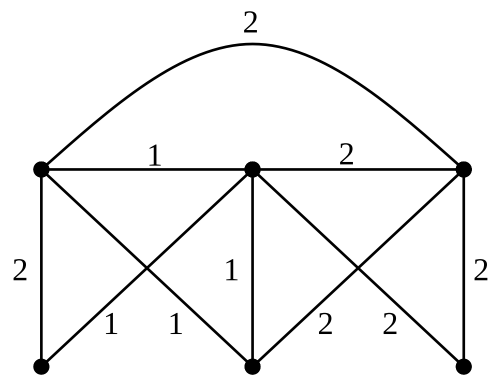


**Figure 9: $(P_3\ \Theta\ K_1)$**

**Illustration 2:**

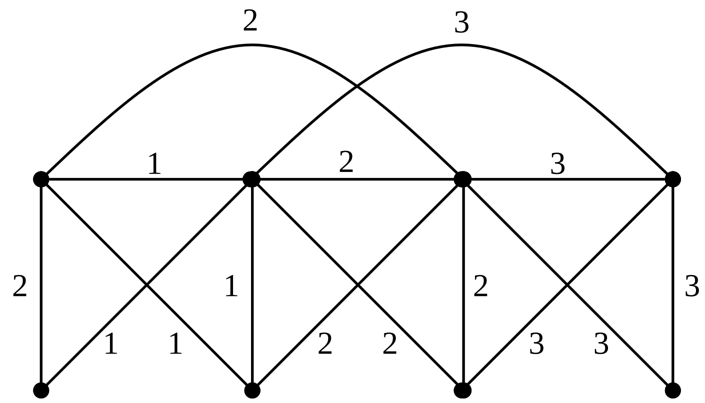


**Figure 10: $(P_4 \Theta K_1)^2$**

**Theorem 4:**

The Moser-Spindle graph which acknowledges rainbow coloring, whose rainbow connection number is 4.

**Proof:**

Consider the Moser Spindle graph with 7 vertices and 11 edges. Let $u_j$ be the vertex set

$0 \leq j \leq 6$.

Then the edge function $f^*$ is defined as follows:

(i) $f^*(u_0 u_1) = 1$

(ii) $f^*(u_1 u_2) = 3$

(iii) $f^*(u_2 u_3) = 2$

(iv) $f^*(u_3 u_4) = 4$

(v) $f^*(u_0 u_4) = 2$

(vi) $f^*(u_0 u_5) = 1$

(vii) $f^*(u_5 u_2) = 3$

(viii) $f^*(u_0 u_6) = 2$

(ix) $f^*(u_6 u_3) = 4$

**Example:**

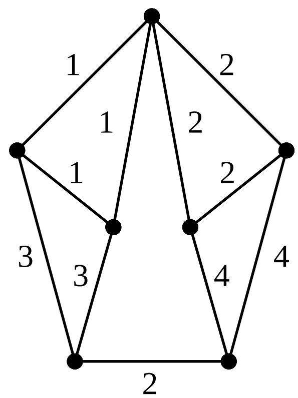


**Figure 11: Moser-Spindle $\gamma_c(G) = 4$**

## Conclusion:

Rainbow connection number for Middle graph of Path Pn, Total graph of path Pn are n+1,n respectively.

Square graph of comb's rainbow connection number is n-1, (n-number of vertices) Moser-Spindle graph's rainbow connection number is 4have been discussed. It is of interest to give Harmonious coloring, greedy coloring etc. for these type of graphs

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