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Research Paper

THE CONNECTED RESTRAINED DETOUR MONOPHONIC DOMINATION NUMBER OF A GRAPH

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Abstract- In this paper the concept of connected restrained detour monophonic domination number $M$of a graph G is introduced. For a connected graph $G = (V, E)$ of order at least two, a connected restrained detour monophonic dominating set M of a graph$G$ is a detour monophonic dominating set such that either $M = V$ or the sub graph induced by $V - M$ has no isolated vertices. The minimum cardinality a connected restrained detour monophonic dominating set of G is the connected restrained detour monophonic domination number of G and is denoted by $\gamma_{dmc_r}(G)$. We determine bounds for it and characterize graphs which realize these bounds. It is shown that For any positive integers r, d and $a \geq 6$ with $r < d$, there exists a connected graph $G$with $rad_m(G) = r$, $diam_m(G) = d$ and $\gamma_{dmc_r}(G) = a$.

Keywords : Minimal detour monophonic dominating set,minimal detour monophonic domination number, restrained detour monophonic dominating set and restrained detour monophonic domination number**.**

I. INTRODUCTION

By a graph G = (V,E) we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by pand q, respectively. The neighborhood of a vertex v of G is the set (v) consisting of all vertices u which are adjacent with v. A vertex v of G is an extreme vertex if the sub graph induced by its neighborhood is complete. A vertex with degree one is called an end vertex. A vertex v of a connected graph G is called a support vertex of G if it is adjacent to an end vertex of G. A vertex v in a connected graph G is a cut vertex of G, if G - vis disconnected. A chord of a path u_1,u_2,u_3,… ,u_k in G is an edge u_i u_j with $j \geq i + 2$. A path P is called a monophonic path if it is a chordless path. A set M of vertices of G is a monophonic set of G if each vertex of G lies on a u-v monophonic path for some u and v in M. A minimalrestrained detour monophonic dominating set of G is a detour monophonic dominating set M such that either $M = V$ or the sub graph induced by $V - M$ has no isolated vertices. The minimum cardinality of a minimalrestrained detour monophonic dominating set of G is the minimalrestrained detour monophonic domination number of G and is denoted by 〖〖γ_dm〗_r〗^+ (G).

Theorem 1.1. [6] Each extreme vertex of a connected graph G belongs to every detour monophonic set of G. Moreover, if the set M of all extreme vertices of G is a detour monophonic set, then M is the unique minimum detour monophonic set of G.

Theorem 1.2. [6] Let G be a connected graph with cut – vertex v and let M be a detour monophonic set of G. Then every component of $G - v$ contains an element of M.

Theorem 1.3. [3] No cut vertex of a connected graph G belongs to any minimum monophonic set of G. Theorem 1.4. [5] Each extreme vertex of a connected graph G belongs to every monophonic dominating set of G.

II. THE UPPER RESTRAINED DETOUR MONOPHONICDOMINATION NUMBER OF A GRAPH

Definition 2.1 A restrained detour monophonic dominating set M in a connected graph G is called a minimal restrained detour monophonic dominating set if no proper subset of M is a restrained detour monophonic dominating set. The maximum counting number in the midst of all minimal restrained detour monophonic dominating set is labeled upper restrained detour monophonic domination number of G, and is denoted by 〖〖γ_dm〗_r〗^+ (G).

Example 2.2 For the graph G given in Figure 5.11, the restrained detour monophonic dominating sets are M_1= {v_1,v_2,v_6,v_7},M_2={v_3,v_4,v_5,v_7 } M_3= {v_1,v_2,v_3,v_5,v_6}. In this graph, the upper restrained detour monophonic domination number is 5 and the restrained detour monophonic domination number is 4.

Theorem 2.3 Each extreme vertex of a connected graph G belongs to every minimal restrained detour monophonic dominating set of G.

Proof. Since every minimal restrained detour monophonic dominating set of G is a restrained detour monophonic dominating set of G, the theorem follows from Theorem 1.2.

Corollary 2.4 For the complete graph K_p, 〖〖γ_dm〗_r〗^+ (K_p)= p.

Theorem 2.5 Let G be a connected graph with cut vertices and let M be a minimal restrained detour monophonic dominating set of G. If v is a cutvertex of G, then every component of G - v contains an element of M.

Proof. Suppose that there is a component B of G-v such that B contains no vertex of M. Let w be a vertex in B. Since M is a minimal restrained detour monophonic dominating set of G, there exist vertices x,y ∈ M such that w lies on some x - y detour monophonic path P: x = u_0,u_1,...,w,...,u_l = y in G. Let P_1 be the x - w subpath of P and P_2 be the w - y subpath of P. Since v is a cutvertex of G, both P_1 and P_2contains v so that P is not a path, which is a contradiction. Thus every component of G - v contains an element of M.

Corollary 2.6 Let G be a connected graph with cut vertices and let M be a minimal restrained detour monophonic dominating set of G. Then every branch of G contains an element of M.

Theorem 2.7 No cut vertex of a connected graph G belongs to any minimal restrained detour monophonic dominating set of G.

Proof. Let M be any minimal restrained detour monophonic dominating set of G and let v ∈ M be any vertex. We claim that, v is not a cutvertex of G. Suppose that v is a cutvertex of G. Let G_1,G_2,...,G_r (r ≥ 2) be the components of G - v. Then v is adjacent to at least one vertex of G_j (1 ≤ j ≤ r). Let M' = M - {v}. Let u be a vertex of G which lies on a detour monophonic path P joining a pair of vertices, say x and v of M. Assume without loss of generality that x ∈ G_1. Since v is adjacent to at least one vertex of each G_j (1 ≤ j ≤ r), assume that v is adjacent to a vertex y in G_k (k = 1). Since M is a minimal restrained detour monophonic dominating set, y lies on a detour monophonic path Q joining v and a vertex z of M such that z (possible y itself) must necessarily belongs to G_1. Thus z = vTheorem 2.8 For any connected graph G, $2 \leq \gamma_{dm_r}(G) \leq \gamma_{dm_r}^{+}(G) \leq p$.

Proof. It is clear from the definition of minimum restrained detour monophonic dominating set that $\gamma_{dm_r}(G) \geq 2$. Since every minimal restrained detour monophonic dominating set is a restrained detour monophonic dominating set of G, $\gamma_{dm_r}(G) \leq \gamma_{dm_r}^{+}(G)$. Also, since V (G) is a restrained detour monophonic dominating set of G, it is clear that $\gamma_{dm_r}^{+}(G) \leq p$. Thus $2 \leq \gamma_{dm_r}(G) \leq \gamma_{dm_r}^{+}(G) \leq p$.

Theorem 2.9 For a connected graph G, $\gamma_{dm_r}(G) = p$ if and only if $\gamma_{dm_r}^{+}(G) = p$.

Proof. Let $\gamma_{dm_r}^{+}(G) = p$. Then M = V (G) is the unique minimal restrained detour monophonic dominating set of G. Since no proper subset of M is a restrained detour monophonic dominating set, it is clear that M is the unique minimum restrained detour monophonic dominating set of G and so $\gamma_{dm_r}(G) = p$. The converse follows from Theorem 2.8.

Theorem 2.10. If G is a connected graph of order pwith $\gamma_{dm_r}(G) = p - 1$, then $\gamma_{dm_r}^{+}(G) = p - 1$. Proof. Since $\gamma_{dm_r}(G) = p - 1$, it follows from Theorem 5.3.4 that $\gamma_{dm_r}^{+}(G) = p$ or $p - 1$. If $\gamma_{dm_r}^{+}(G) = p$, then by Theorem 2.9, $\gamma_{dm_r}(G) = p$, which is a contradiction. Hence $\gamma_{dm_r}^{+}(G) = p – 1$.

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