e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.12, 2022
  UGC CARE Listed (Group -I) Journal

Group Mean Cordial Labeling of Triangular Snake Related Graphs

R.N.RAJALEKSHMI, *Research Scholar, Department of Mathematics, ManonmaniamSundaranar University, Abishekapatti, Tirunelveli, India,*[1]rajalekshmimoni@gmail.com

R.KALA, *Department of Mathematics, ManonmaniamSundaranar University, Abishekapatti, Tirunelveli, India,* arthipyi91@gmail.com

ABSTRACT — Let G be a (p, q) graph and let A be a group. Let $g : V(G) \to A$ be a map. For each edge xy assign the label $\left\lfloor \frac{o(g(x))+o(g(y))}{2} \right\rfloor$. Here $o(g(x))$ denotes the order of $g(x)$ as an element of the group A. Let I be the set of all integers that are labels of the edges of G. g is called a group mean cordial labeling if the following conditions hold:
(1) For a, b ∈ A, $|v_g(a) - v_g(b)| \leq 1$, where $v_g(a)$ is the number of vertices labeled with a.
(2) For r, s ∈ I, $|e_g(r) - e_g(s)| \leq 1$, where $e_g(r)$ denote the number of edges labeled with r.
A graph with a group mean cordial labeling is called a group mean cordial graph. In this paper, we take A as the group of fourth roots of unity and prove that, Triangular snake, Double triangular snake and Alternate triangular snake are group mean cordial graphs.

Keywords — Cordial labeling, mean labeling, group mean cordial labeling,Triangular snake, Double triangular snake, Alternate triangular snake.

AMS Subject Classification— 05C78.

I. INTRODUCTION

Graphs considered here are finite, undirected and simple. Terms not defined here are used in the sense of Harary and Gallian [3]. Somasundaram and Ponraj introduced the concept of mean labeling of graphs.

Definition 1.1. [6] A graph G with p vertices and q edges is a mean graph if there is an injective function g from the vertices of G to 0, 1, 2, ..., q such that when each edge xy is labeled with $\frac{g(x)+g(y)}{2}$ if g(x) + g(y) is even and $\frac{g(x)+g(y)+1}{2}$ if g(x) + g(y) is odd then the resulting edge labels are distinct. Cahit [2] introduced the concept of cordial labeling.

Definition 1.2. [2] Let g : V (G) → {0, 1} be any function. For each edge uv assign the label |g(u) − g(v)|. g is called a cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1. Also the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1.

Ponraj et al. [5] introduced mean cordial labeling of graphs.

Definition 1.3. [5] Let g be a function from the vertex set V (G) to {0, 1, 2}. For each edge xy assign the label l $\left\lceil \frac{g(x)+g(y)}{2} \right\rceil$. g is called a mean cordial labeling if $|v_g(r) - v_g(s)| \leq 1$ and $|e_g(r) - e_g(s)| \leq 1$ r, s ∈ {0, 1, 2}, where $v_g(u)$ and $e_g(u)$ respectively denote the number of vertices and edges labeled with u (u = 0, 1, 2). A graph with a mean cordial labeling is called a mean cordial graph.

Athisayanathan et al. [1] introduced the concept of group A cordial labeling.

II. MAIN RESULTS

Definition 2.1. Let G be a (p, q) graph and let A be a group. Let $g : V(G) \to A$ be a map. For each edge xy assign the label $\left\lfloor \frac{o(g(x))+o(g(y))}{2} \right\rfloor$. Here $o(g(x))$ denotes the order of $g(x)$ as an element of the group A. Let I be the set of all integers that are labels of the edges of G. g is called a group mean cordial labeling if the following conditions hold:

(1) For a, b ∈ A, $|v_g(a) - v_g(b)| \leq 1$, where $v_g(a)$ is the number of vertices labeled with a.
(2) For r, s ∈ I, $|e_g(r) - e_g(s)| \leq 1$, where $e_g(r)$ denote the number of edges labeled with r.
A graph with a group mean cordial labeling is called a group mean cordial graph.

In this paper, we take the group A as the group {1, −1, i, −i} which is the group of fourth roots of unity, that is cyclic with generators i and −i.

Theorem 2.1. The Triangular Snake graph T_n is a group mean cordial graph for every n.
Proof. Let $P_n = x_1x_2 \ldots x_n$ be a path. Let $V(T_n) = V(P_n) \cup \{y_j: 1 \leq j \leq n-1\}$. Then $E(T_n) = E(P_n) \cup \{x_jy_j, x_{j+1}y_j: 1 \leq j \leq n-1\}$. The order and size of T_n are 2n-1 and 3n-3 .
Case 1: $n \equiv 0,1,2 \pmod 4$.
Define $g: V(T_n) \rightarrow \{1, -1, i, -i\}$ by,

$$g(x_j) = \begin{cases} -1 & if\ j \equiv 1\ (mod\ 4) \\ i & if\ j \equiv 2\ (mod\ 4) \\ -i & if\ j \equiv 3\ (mod\ 4) \\ 1 & if\ j \equiv 0\ (mod\ 4) \end{cases}$$

and

$$g(y_j) = \begin{cases} 1 & if\ j \equiv 1\ (mod\ 4) \\ i & if\ j \equiv 2\ (mod\ 4) \\ -1 & if\ j \equiv 3\ (mod\ 4) \\ -i & if\ j \equiv 0\ (mod\ 4) \end{cases}$$

Case 2: $n \equiv 3 \pmod 4$.
The group mean cordial labeling of T_3 is given in Fig.2.2.
Assign the labels as in case 1 to the vertices x_j ($1 \leq j \leq$ n-3) and y_j ($1 \leq j \leq$ n-4) .
Next label x_{n-2}, x_{n-1}, x_n as $-1, i, 1$ in order and $y_{n-3}, y_{n-2}, y_{n-2}$ as $-1, -i, -i$ in order.
The values of v_g (j) and e_g(s) are tabulated in Tables 2.1 and 2.2.

Nature of n	v_g ***(1)***	v_g***(-1)***	v_g ***(i)***	v_g***(-i)***
$n \equiv 0,2$ (mod 4)	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2} - 1$
$n \equiv 1,3$ (mod 4), $n \neq 3$	$\frac{n-1}{2}$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n+1}{2}$

TABLE 2.1.

Hence Table. prove that g is a group mean cordial labeling.

Theorem 2.2. Double Triangular Snake graph $D(T_n)$ is a group mean cordial graph for every n.
Proof. Let $P_n = x_1x_2 \ldots x_n$ be the common path. Let a_j, b_j ($1 \leq j \leq n-1$) be the newly added vertices.Then E(D(T_n)) =E(P_n) ∪ $\{x_ja_j, x_{j+1}a_j, x_jb_j, x_{j+1}b_j : 1 \leq j \leq n-1\}$. The order and size of D (T_n) are 3n-2 and 5n-5.Define $g: V(T_n) \rightarrow \{1, -1, i, -i\}$ by,

$g(x_j) = \begin{cases} 1 & if\ j \equiv 1\ (mod\ 4) \\ -1 & if\ j \equiv 2\ (mod\ 4) \\ i & if\ j \equiv 3\ (mod\ 4) \\ -i & if\ j \equiv 0\ (mod\ 4) \end{cases}$ and $g(a_j) = g(b_j) = \begin{cases} i & if\ j \equiv 1\ (mod\ 4) \\ 1 & if\ j \equiv 2\ (mod\ 4) \\ -i & if\ j \equiv 3\ (mod\ 4) \\ -1 & if\ j \equiv 0\ (mod\ 4) \end{cases}$

Case 2: $n \equiv 2\ (mod\ 4)$. Assign the labels to the vertices $x_j(1 \le j \le n-1)$ *and* $a_j, b_j(1 \le j \le n-2)$ as in case 1.Then assign $i, -1, -i$ to the vertices x_n, a_{n-1}, b_{n-1} in order. *Case 3:* $n \equiv 3\ (mod\ 4)$.
Assign the labels to the vertices $x_j(1 \le j \le n-2)$ *and* $a_j, b_j(1 \le j \le n-3)$ as in case 1. Next label a_{n-2}, b_{n-2} with -1 ; x_{n-1}, a_{n-1} with i ; x_n with $-i$ and b_{n-1} with 1.Case *4:* $n \equiv 0\ (mod\ 4)$.
Assign the labels to the vertices $x_j(1 \le j \le n-3)$ *and* $a_j, b_j(1 \le j \le n-4)$ as in case 1. Next assign i to the vertices x_{n-2}, a_{n-2} ; -1 to the vertices $a_{n-3}, b_{n-3}, b_{n-2}$; $-i$ to the vertices x_{n-1}, a_{n-1} and assign 1 to the vericesxx_n, b_{n-1}.
Tables 2.3 & 2.4 prove that g is a group mean cordial labeling.

Nature of n	v_g ***(1)***	v_g ***(-1)***	v_g ***(i)***	v_g ***(-i)***
$n \equiv 0$ (mod 4)	$\frac{3n}{4}$	$\frac{3n}{4}$	$\frac{3n}{4} - 1$	$\frac{3n}{4}$
$n \equiv 1$ (mod 4)	$\frac{3n+1}{4}$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$
$n \equiv 2$ (mod 4)	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$
$n \equiv 3$ (mod 4)	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n-5}{4}$

TABLE 2.3.

Theorem 2.3. The Alternate Triangular Snake A(T_n)is a group mean cordial graph when n is odd.
Proof. Let $P_n = x_1 x_2 \ldots x_n$ be a path.
Case 1: The Alternative Triangular snake starts with triangle.
Let $V(A(T_n)) = V(P_n) \cup \{y_j : 1 \le j \le \frac{n-1}{2}\}$. Then $E(A(T_n)) = E(P_n) \cup \{x_j y_{\frac{j+1}{2}} : j \equiv 1\ (mod\ 2) \cup$ $\{x_j y_{\frac{j}{2}} : j \equiv 0\ (mod\ 2)\}$.
The order and size of this graph are $\frac{3n-1}{2}$ and 2n-2.
Subcase 1.1: $n \equiv 1\ (mod\ 8)$.
Define $g: V(A(T_n)) \to \{1, -1, i, -i\}$ by,

$g(x_j) = \begin{cases} 1 & if\ j \equiv 0,1,2\ (mod\ 8) \\ -1 & if\ j \equiv 4,5\ (mod\ 8) \\ i & if\ j \equiv 3,6,7\ (mod\ 8) \end{cases}$ and $g(y_j) = \begin{cases} -1 & if\ j \equiv 1\ (mod\ 4) \\ -i & if\ j \equiv 0,2,3\ (mod\ 4) \end{cases}$

Subcase 1.2: $n \equiv 3\ (mod\ 8)$.
Label the vertices $x_j(1 \le j \le n-2), y_j(1 \le j \le \frac{n-1}{2})$as in subcase 1.1. Then label x_{n-1} with i and y_n with $-i$.

Subcase 1.3: $n \equiv 5\ (mod\ 8)$.
Label the vertices $x_j(1 \le j \le n-4), y_j(1 \le j \le \frac{n-5}{2})$ as in subcase 1.1. Next define $g(x_{n-3}) = 1$; $g(x_{n-2}) = -1$; $g(x_{n-1}) = g(x_n) = i$ and $g\left(y_{\frac{n-3}{2}}\right) = g\left(y_{\frac{n-1}{2}}\right) = -i$.

REFERENCES

[1] S.Athisayanathan, R.Ponraj, and M. K.Karthik Chidambaram, Group A cordial labeling of Graphs,

International Journal of Applied Mathematical Sciences, Vol 10, No.1(2017),1-11 .
[2] I.Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, *ArsCombin*., 23(1987), 201-207.
[3] J. A.Gallian, A Dynamic survey of Graph Labeling, *The Electronic Journal of Combinatorics*, Dec 7(2015), No. DS6.
[4] F.Harary, *Graph Theory*, Addison Wesley, Reading Mass, 1972.
5] R.Ponraj, M.Sivakumar, M.Sundaram, Mean cordial labeling of graphs, Open *Journal of Discrete Mathematics*, No.2(2012), 145-148.
[6] S. Somasundaram and R. Ponraj , Mean labeling
of graphs, *Natl.Acad.Sci.Let.* 26(2003), 210-213.