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A METHOD FOR PARTIAL DIFFERENTIAL EQUATION USING COMPUTER ALGEBRA

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ABSTRACT: In mathematics and computer science, computer algebra, also called symbolic computation or algebraic computation, is a scientific area that refers to the study and development of algorithms and software for manipulating mathematical expressions and other mathematical objects. Mathematics and computer science, computer algebra, also called symbolic computation or algebraic computation, is a scientific area that refers to the study and development of algorithms and software for manipulating mathematical expressions and other mathematical objects. Symbolic computation techniques for solutions of Partial Differential Equations (PDEs) with Maxima, an open source Computer Algebra System (CAS) are represented in three new packages. They are prepared in Maxima's user's language. After giving the initial and boundary conditions, packages automatically give the results and plot the graphs. The packages are designed as ready to use and modify instructional tools by users with computer-lab sessions of engineering courses. Solutions are performed by Finite Difference Method (FDM). Hence, this model will shows better results interms of reducing computing time and increases accuracy.

Keywords: Partial Differential Equations, Numerical Methods, Finite Difference Method, Computer Algebra, Maxima

I. INTRODUCTION

A Computer Algebra System (CAS) solves, plots, and manipulates mathematical expressions in an analytical form. CASs support a wide range of mathematics such as linear algebra, calculus, and algebraic and ordinary differential equations. Computer algebra systems can represent parameters and variables in equations as symbols [1].

Using a CAS you can solve, manipulate, and plot f(x) without having to generate numeric data.

You can perform sensitivity studies to learn how an equation changes due to its parameters by easily substituting numerical values for symbolic parameters and variables.

Computer algebra is that part of computer science which designs, analyzes, implements and applies algebraic algorithms. We may consider this as a working definition for the scope of the following chapters but we do not claim that this is the only possible one. The definition is based on the view that the study of algorithms is at the heart of computer science [2]. Compared with other algorithms, algebraic algorithms have simple formal specifications, have proofs of correctness and asymptotic time bounds which can be established on the base of a well developed mathematical theory. Furthermore, algebraic objects can be represented exactly in the memory of a computer, so that algebraic computations can be performed without loss of precision and significance.

Usually, algebraic algorithms are implemented in software systems allowing input and output in the symbolic notation of algebra. For all these reasons, computer algebra has attracted an increasing interest in computer science, mathematics, and application areas [3]. Negatively, one could say that computer algebra treats those subjects which are too computational

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to appear in an algebra text and which are too algebraic to be presented in a usual computer science text book. Another negative description says that computer algebra deals with non-numeric computations, a description which had only some content in the very early days of computing. Somewhat in between is Knuth's definition of semi numerical algorithms.

PDEs are used to describe many mathematical problems such as diffusion, wave, Laplace's and Poisson's equations in science and engineering. PDEs can be solved with analytically in a very few closed-forms. Numerical methods are commonly used to solve them. They can be grouped in two wide groups; finite element and finite difference methods. Both of these methods transform a PDE problem to the problem of solving of algebraic equation system [4]. However, numerical solutions are time consuming and exhausting. Computer algebra packages ease this long process.

CASs are very helpful tools in the solution of time consuming and exhausted engineering problems. These software systems affect the way of learning-teaching math and encourage the scientists for the investigations in theoretical and practical branches of science. 'Matlab®', 'Maple®' and 'Mathematica®' that are being developed by companies according to the needs of education, engineering and technology are popular, integrated and general purpose CASs including special packages for special purposes. There are open source and open code CAS, too, such as 'Maxima', 'Sage' and 'Scilab'. These systems are being developed by the volunteers around the world [5]. 'Maxima' is one of the most famous general-purpose CAS. It has a symbolic and very powerful math-engine 'as much as Maple'.

Sample studies performed with CASs avoid them from being a 'black box' and encourage the users to use the CAS. What can be done with a CAS that has much more abilities than a calculator is important as much as what it presents to its users like instructors. Kuzler declared in his research showing the sub-steps of the solutions, enabling additional applications within a study by changing the inputs, plotting graphs, obeying the paper-pen solution algorithms in worksheets and having a user-friendly userlanguage make the CAS more attractive for the nonprogrammer instructors. Some sample studies with 'Maxima' according to the proposals declared above have presented [6]. There are lots of available studies done with proprietary CASs. CASs include "formulae Cruncher" commands. However, the solution of PDEs cannot be executed with only one command. The solutions of PDEs contain many steps. FDM packages in FORTRAN language, with Matlab's user language are available. Such attempts are done with Maple and Matematica by their users. In this paper, three new Maxima '.mac' files are presented performing the solutions of PDEs with a numeric method FDM.

II. LITERATURE SURVEY

T. Şirin., et.al [7] Symbolic computation techniques for solutions of Partial Differential Equations (PDEs) with Maxima, an open source computer algebra system (CAS) are represented in three new packages. They are prepared in Maxima's user's language. After giving the initial and boundary conditions, packages automatically give the results and plot the graphs. The packages are designed as ready to use and modify instructional tools

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by users with computer-lab sessions of engineering courses. Solutions are performed by Finite Difference Method (FDM).

V. Y. Glizer and V. Turetsky., et.al [8] A finite-horizon linear-quadratic optimal control problem for systems with point-wise and distributed state delays is considered. Based on well known control optimality conditions, the solution of this problem is reduced to solution of the set of three Riccati-type matrix differential equations: an ODE and two first-order PDEs with two and three independent variables. In the paper, this set of Riccati-type equations is further reduced to a set of two equations. One of these equations is the ODE, while the other is a partial integro-differential equation. A numerical procedure of solution of this new set of equations is proposed. An illustrative example is presented.

X. Xu and R. J. Adams., et.al [9] Overlapped, localizing local-global solution (OL-LOGOS) modes have been proposed to develop fast direct solvers for low frequency electromagnetic wave problems. The efficiencies of the resulting OL-LOGOS factorization algorithms have been demonstrated for the matrix equations associated with dense three-dimensional integral equations and sparse two-dimensional partial differential equations. In both cases, approximately O(N log N) time and O(N) memory complexities have been observed. In this work, the OL-LOGOS method is applied to three-dimensional scalar FEM systems. In order to improve the factorization speed and reduce memory costs for FEM applications, a pre-factorization permutation step is incorporated into the OL-LOGOS factorization algorithm. Numerical results demonstrate factorization and memory complexities of approximately O(N log N) and O(N) as the problem size grows.

H. Zhang, C. Zhu, Q. Peng and J. X. Chen, et.al [10] different methods exist for deriving a 3D linear transformation, including vector algebra, geometric algebra, and matrix algebra (Artin, 1991; Hestenes, 1986, Foley et al., 2002). In this paper, we introduce Ron Goldman's unified derivation method in vector algebra (Goldman, 2003) combined with our developments in geometric algebra (Hestenes, 1986; Dorst and Mann, 2002; Mann and Dorst, 2002). Our work includes using a unified method to derive various geometrical transformations, which are different from the current ray-tracing and transformation methods implemented in graphics algorithms and hardware (Foley et al., 2002). We hope that these derivations extend the geometric algebra research and applications to the computer graphics field.

B. Erabadda, S. Ranathunga and G. Dias, et.al [11] presents a system that automatically assesses multi-step answers to algebra questions. The system requires teacher involvement only during the question set-up stage. Two types of algebra questions are currently supported: questions with linear equations containing fractions, and questions with quadratic equations. The system evaluates each step of a student's answer and awards full/partial marks according to a marking scheme. The system was evaluated for its performance using a set of student answer scripts from a government school in Sri Lanka and also by undergraduate students. The system accuracy was over 95.4%, and over 97.5%, respectively for the aforementioned data sets.

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F. Zeynivandnezhad, Z. Ismail and Y. M. Yusof, et.al [12] Engineers often need to make sense of mathematical structures in differential equations to deal with real life problems. However, students showed difficulties in grasping them. This study presents the teaching of mathematical structures in differential equations using a computer algebra system. The computer algebra systems played the role as a pedagogical tool to enhance teaching mathematical structures. Findings showed the nature of the problems, prompts and questions assisted engineering undergraduates to make sense of mathematical structures. Explanations and links were the most frequent type of mathematical structures employed in solving real life problems. While techniques and representations were the most frequent mathematical structures adopted during solving procedural problems. This study implies that instructors should use computer algebra systems strategically to teach meaningful mathematics.

F. -a. Deng, T. Chen and S. Ren, et.al [13] The main aim of this article is to study Wd -fuzzy implication algebras which are subalgebra of fuzzy implication algebras. We showed that Wd -fuzzy implication algebras are regular fuzzy implication algebras, but the inverse is not true. The relations between Wd -fuzzy implication algebras and other fuzzy algebras are discussed. Properties and axiomatic systems for Wd -fuzzy implication algebras are investigated. Furthermore, a few new results on Wd -fuzzy implication algebras has been added.

F. -a. Deng, T. Chen and S. Ren, et.al [14] Geometric or Clifford Algebra (CA) is a powerful mathematical tool that is attracting a growing attention in many research fields such as computer graphics, computer vision, robotics and medical imaging for its natural and intuitive way to represent geometric objects and their transformations. This paper introduces the architecture of CliffordCoreDuo, an embedded dual-core coprocessor that offers direct hardware support to four-dimensional (4D) Clifford algebra operations. A prototype implementation on an FPGA board is detailed. Experimental results show a 1.6x average speedup of Clifford Core Duo in comparison with the baseline mono-core architecture. A potential cycle speedup of about 40x over Gaigen 2, a geometric algebra software library generator for general-purpose processors, is also demonstrated

S. Franchini, A. Gentile, G. Vassallo, F. Sorbello and S. Vitabile, et.al [15] This paper presents a system that automatically identifies errors made by students in answering algebra questions that require multiple steps. The types of algebra questions we consider include linear equations with fractions and quadratic equations. We have already developed a system that is capable of grading multi-step answers to the aforementioned two types of questions and awarding full/partial credit according to a marking scheme. The error identification module works on top of this previous system. It was evaluated using data from two sources: government schools and a tuition class in Sri Lanka. The mistakes identified by the system were compared against feedback by two independent teachers. The results showed that the system identified the student mistakes with more than 85% accuracy for both types of questions.

III. METHODOLOGY

A computer algebra system (CAS) solves, plots, and manipulates mathematical

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expressions in an analytical form. CASs support a wide range of mathematics such as linear algebra, calculus, and algebraic and ordinary differential equations. Computer algebra systems can represent parameters and variables in equations as symbols like $f(x)=1-ax^2$. Using a CAS you can solve, manipulate, and plot f(x) without having to generate numeric data. You can perform sensitivity studies to learn how an equation changes due to its parameters by easily substituting numerical values for symbolic parameters and variables. Using the Live Editor in MATLAB®, symbolic calculations look like the equations. MATLAB supports both numeric and symbolic approaches to mathematical modeling, which lets you solve problems using the best approach. CASs have sophisticated algorithms for solving and simplifying algebraic equations, systems of equations, and systems of Differential Algebraic Equations (DAEs).

To solve a linear parabolic PDE, which serves as a continuous model e.g., to describe diffusion, we need to use some numerical methods. The same situation appears when we need to determine the equilibrium state of the same parabolic PDE, which is equivalent to solving the corresponding elliptic PDE, namely, some numerical method is necessary. In this manner, we arrive at a discrete problem, which can be solved numerically on a computer. Continuous mathematical models of a real-life problem have different qualitative properties, e.g., the diffusion equation possesses different maximum principles, the non-negativity preservation property, the stabilization property, etc.

FDM for parabolic PDEs with 'Maxima'.
"ParaPDE.mac"

```
--> declare([r,iend,jend,fini],constant);
 --> r:0.5;iend:20.0;jend:20;fini:100.0;
--> for j:1.0 thru jend do for i:1.0 thru iend
        do f[i,j]:fini; --> for j:1.0 thru jend
        do f[1.0,j]:0.0;
--> for i:iend thru iend do for j:1.0 thru
        jend do f[iend,j]:0.0;
--> f[1.0,1.0]:fini/2.0;f[iend,1.0]:fini/2.0;
--> f[i,j]:for j:1.0 thru jend-1.0 do for i:2.0
        thru iend-1.0 do      (f[i,j+1.0]: r*
        (f[i+1.0,j]+f[i-1.0,j])+(1-2*r)
        *f[i,j]);
-->load (draw) $ >M:apply(matrix,make
        list(make list (f[i,j],j,1.0,19.0)
        ,i,1.0,19. 0 )); draw3d (contour_
        levels=10,20,30,40,50,60,70,80,90,
        100},contour = both, color = blue,
        elevation_grid(M,0,0,1,1),xlabel =
        "x",ylabel = "y",surface_hide =
        true);
-->im: apply (matrix, make list (make list
        (f[i,j],j,1.0,19.0), i,1.0,19.0)) $
→Draw 2d (palette = gray, image
        (im,0,0,100,100))$
```

IV. RESULT ANALYSIS

In this section result analysis of differential equations with computer algebra is observed in this section.

Table.1: Performance Analysis

Parameter	Fuzzy	FDM
Accuracy	89	96
Computing Time	9785	7956

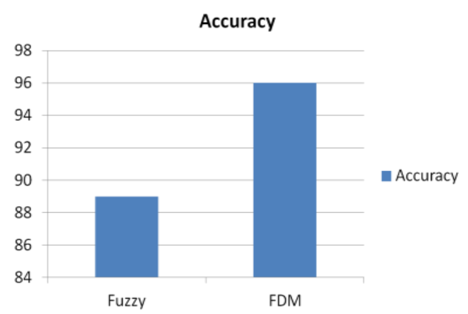

Fig1: Accuracy Comparison Graph

In Fig.1 accuracy comparison graph is seen between Fuzzy and FDM.

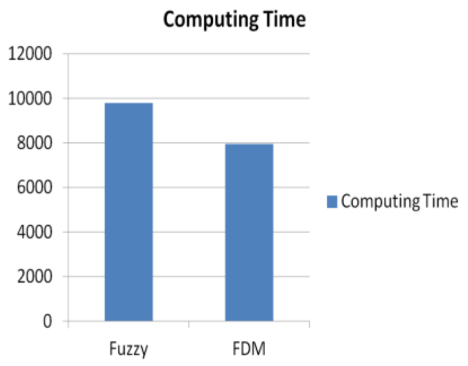

Fig.2: Computing Time Comparison Graph

In Fig.2 computing time comparison graph is seen between Fuzzy and FDM.

V. CONCLUSION

Techniques for symbolic computing of PDEs using Maxima are demonstrated. A sample project to create new Maxima packages for related research is offered to teachers and engineering course participants who are not programmers. Maxima is an open-source computer algebra system that has demonstrated its ability to program and solve time-consuming PDE problem solutions. Hence, this system achieves better results interms of accuracy and time saving.

VI. REFERENCES

[1] S. Dost and others, “Use of Computer Algebra Systems in Mathematics Teaching at University: a Teaching Experiment” H. U. Journal of Education 40: 140-151, 2011.

[2] D. Picardi, “Motivating Constraints of Embedded Computer Algebra System”, 2006. Retrieved on 15 May 2011 from http://homedir.jct.ac.il/~sifriah/reprints/dana-2007.pdf

[3] T. Sirin, “Finite Difference Method Approximations in 2-D Electrostatics”, 7th International Conference on Electronics and Computer Engineering, 1-2 November 2010- Bishkek.

[4] T. Sirin,“Symbolic Computation of Second Order Differential Equations with Finite Difference Method with Maxima”, 7th International Conference on Electronics and Computer Engineering, 1-2 November 2010- Bishkek

[5] T. Sirin, V.F. Babak, “Programming with Maxima”, Conference of Information Technologies in Education Status, Problems and Perspectives, - Bishkek, 1-2 July 2011 .

[6] N, Ari, , S.Özen, “Elektomanyetik Alanlar Teori ve Matlab Uygulamaları” Palme Yayincilik pp.1-16, 2008

[7] T. Şirin, "Solutions of partial differential equations with computer algebra," *2013 International Conference on Electronics, Computer and Computation (ICECCO)*, Ankara, Turkey, 2013, pp. 351-354, doi: 10.1109/ICECCO.2013.6718300.

[8] V. Y. Glizer and V. Turetsky, "Optimal quadratic control of linear time delay systems: One approach to numerical solution," *11th IEEE International Conference on Control & Automation (ICCA)*, Taichung, Taiwan, 2014, pp. 797-802, doi: 10.1109/ICCA.2014.6871023.

[9] X. Xu and R. J. Adams, "Fast direct solution of FEM systems using overlapped localizing modes on a shifted grid," *2011 IEEE International Symposium on Antennas and Propagation (APSURSI)*, Spokane, WA, USA, 2011, pp. 3265-3267, doi: 10.1109/APS.2011.5997231.

[10] H. Zhang, C. Zhu, Q. Peng and J. X. Chen, "Using geometric algebra for 3D linear transformations," in *Computing in Science & Engineering*, vol. 8, no. 3, pp. 68-75, May-June 2006, doi: 10.1109/MCSE.2006.54.

[11] B. Erabadda, S. Ranathunga and G. Dias, "Computer Aided Evaluation of Multi-Step Answers to Algebra Questions," *2016 IEEE 16th International Conference on Advanced Learning Technologies (ICALT)*, Austin, TX, USA, 2016, pp. 199-201, doi: 10.1109/ICALT.2016.35.

ISSN PRINT 2319 1775 Online 2320 7876

Research paper Volume 10, Iss 4, 2021

[12] F. Zeynivandnezhad, Z. Ismail and Y. M. Yusof, "Teaching mathematical structures in differential equations using a computer algebra system to engineering students," *2015 IEEE 7th International Conference on Engineering Education (ICEED)*, Kanazawa, Japan, 2015, pp. 10-15, doi: 10.1109/ICEED.2015.7451483.

[13] F. -a. Deng, T. Chen and S. Ren, "On Wd-Fuzzy Implication Algebras," *2017 4th International Conference on Information Science and Control Engineering (ICISCE)*, Changsha, China, 2017, pp. 510-513, doi: 10.1109/ICISCE.2017.113.

[14] S. Franchini, A. Gentile, G. Vassallo, F. Sorbello and S. Vitabile, "A Dual-Core Coprocessor with Native 4D Clifford Algebra Support," *2012 15th Euromicro Conference on Digital System Design*, Cesme, Turkey, 2012, pp. 419-422, doi: 10.1109/DSD.2012.2.

[15] B. Erabadda, S. Ranathunga and G. Dias, "Automatic Identification of Errors in Multi-step Answers to Algebra Questions," *2017 IEEE 17th International Conference on Advanced Learning Technologies (ICALT)*, Timisoara, Romania, 2017, pp. 215-219, doi: 10.1109/ICALT.2017.66