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On Radio Heronian Mean Dd-Distance Of Some Operation Graphs

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ABSTRACT

A Radio Heronian Mean Dd-distance Labeling of a connected graph G is an injective map f from the vertex set $V(G)$ to the N such that for two distinct vertices u and v of G, $D^{Dd}(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^{Dd}(G)$ where $D^{Dd}(u,v)$ denotes the Dd-distance between u and v and $diam^{Dd}(G)$ denotes the Dd-diameter of G. The radio heronian Dd-distance number of f, $rhmn^{Dd}(f)$ is the maximum label assigned to any vertex of G. The radio heronian Dd-distance number of G, $rhmn^{Dd}(G)$ is the minimum value of $rhmn^{Dd}(f)$ taken over all radio heronian Dd-distance labeling f of G. We study on in this paper about on radio heronian mean Dd-distance of some operation graphs.

Keywords: *Radio heronian mean Dd-distance number, crown graph, comb graph, ladder graph and bistar.*

1. INTRODUCTION

A graph $G = (V, E)$ we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. Graph labeling was introduced by Alexander Rosa in 1967. Radio mean labeling was introduced by S. Somasundaram and R. Ponraj in 2004. Harmonic mean labeling was introduced by S. Somasundaram and S S Sandhya in 2012.

The concept of D-distance was introduced by D. Reddy Babu et al. The concept of radio D-distance was introduced by T. Nicholas and K. John Bosco in 2017. The concept of radio

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mean D-distance was introduced by T. Nicholas and K. John Bosco in 2017. The concept of heronian mean labeling was introduced by S S Sandhya in 2017. The Dd-distance was introduced by A. Anto kinsely and P. Siva Ananthi in 2017. The concept of radio heronian mean Dd- distance labeling of some basic graphs was introduced by K. John Bosco and Dinesh M in 2021. For a connected graph G, $u - v$ path is defined as $D^{Dd}(u,v) = D(u,v) + \deg(u) + deg(v)$.

We are introducing the concept of radio heronian mean Dd-distance of some operation graphs. A Radio Heronian Mean Dd-distance Labeling of a connected graph G is an injective map f from the vertex set $V(G)$ to the N such that for two distinct vertices u and v of G, $D^{\text{Dd}}(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^{\text{Dd}}(G)$ where $D^{\text{Dd}}(u,v)$ denotes the Dd-distance between u and v and $diam^{\text{Dd}}(G)$ denotes the Dd-diameter of G. The radio heronian Dd-distance number of f, $rhmn^{\text{Dd}}(f)$ is the maximum label assigned to any vertex of G. The radio heronian Dd-distance number of G, $rhmn^{\text{Dd}}(G)$ is the minimum value of $rhmn^{\text{Dd}}(f)$ taken over all radio heronian Dd-distance labeling f of G.

2. Main Results

Theorem: 2.1

The radio heronian mean Dd-distance number of a crown graph,

$$rhmn^{D}(C_n \odot K_1) = \begin{cases} 5\left(\frac{n-1}{2}\right) + 2, if\ n\ is\ odd\ (\ n \geq 3) \\ 5\left(\frac{n}{2}\right) - 1, if\ n\ is\ even\ (\ n \geq 4) \end{cases}$$

Proof:

Let $\{v_1, \dots \dots . . v_n\}$ and $\{u_1, u_2, \dots \dots . . u_n\}$ are the vertex set, where $\{u_1, u_2, \dots \dots . . u_n\}$ are the pendent vertices.

So $diam^{\text{Dd}}\,(C_n \odot K_1) = n + 5$

Without loss of generality, let $f(v_1) < f(v_2) < \cdots \dots \dots \dots \dots . < f(v_{n-1})$.

We shall check the radio heronian mean Dd-distance condition

$$D^{\text{Dd}}(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^{\text{Dd}}(G)$$

If n is odd, then

$D^{Dd}(v_1,u_1) = D^{Dd}(v_2,u_2) = D^{Dd}(v_3,u_3) = n + 2, D^{Dd}(v_1,u_2) = n + 3, 1 \leq i \leq n$

$D^{Dd}(u_1,u_2) = n + 2\ , D^{Dd}(v_1,v_2) = n + 4, \quad 1 \leq i \leq n$

If (v_1,u_1) and (v_1,v_2) are adjacent,

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$$D^{Dd}(v_1,u_1)+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(u_1)}+f(u_1)}{3}\right\rceil \geq n+2+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(u_1)}+f(u_1)}{3}\right\rceil \geq n+6$$

$$D^{Dd}(v_1,v_2)+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3}\right\rceil \geq n+4+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3}\right\rceil \geq n+6$$

If (v_1,u_2) and (u_1,u_2) are non adjacent,

$$D^{Dd}(v_1,u_2)+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(u_2)}+f(u_2)}{3}\right\rceil \geq n+3+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3}\right\rceil \geq n+6$$

$$D^{Dd}(u_1,u_2)+\left\lceil\frac{f(u_1)+\sqrt{f(u_1)f(u_2)}+f(u_2)}{3}\right\rceil \geq n+2+\left\lceil\frac{f(u_1)+\sqrt{f(u_1)f(u_2)}+f(u_2)}{3}\right\rceil \geq n+6$$

Therefore $f(v_i)=\left(\frac{n-1}{2}\right)+i, 1 \leq i \leq n$

If n is even, then

$D^{Dd}(v_1,u_1)=D^{Dd}(v_2,u_2)=D^{Dd}(v_3,u_3)=n+1, D^{Dd}(v_1,u_2)=n+2, 1 \leq i \leq n$

$D^{Dd}(u_1,u_2)=n+1\,, D^{Dd}(v_1,v_2)=n+3, \quad 1 \leq i \leq n$

If (v_1,u_1) and (v_1,v_2) are adjacent,

$$D^{Dd}(v_1,u_1)+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(u_1)}+f(u_1)}{3}\right\rceil \geq n+1+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(u_1)}+f(u_1)}{3}\right\rceil \geq n+6$$

$$D^{Dd}(v_1,v_2)+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3}\right\rceil \geq n+3+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3}\right\rceil \geq n+6$$

If (v_1,u_2) and (u_1,u_2) are non adjacent,

$$D^{Dd}(v_1,u_2)+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(u_2)}+f(u_2)}{3}\right\rceil \geq n+2+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3}\right\rceil \geq n+6$$

$$D^{Dd}(u_1,u_2)+\left\lceil\frac{f(u_1)+\sqrt{f(u_1)f(u_2)}+f(u_2)}{3}\right\rceil \geq n+1+\left\lceil\frac{f(u_1)+\sqrt{f(u_1)f(u_2)}+f(u_2)}{3}\right\rceil \geq n+6$$

Therefore $f(v_i)=\left(\frac{n}{2}\right)+i-1, 1 \leq i \leq n$

Hence $rhmn^D(C_n \odot K_1)=\begin{cases}5\left(\frac{n-1}{2}\right)+2, if\ n\ is\ odd\ (n \geq 3)\\ 5\left(\frac{n}{2}\right)-1, if\ n\ is\ even\ (n \geq 4)\end{cases}$

Theorem 2.2

The radio heronian mean Dd-distance number of a K_2+mK_1,

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$$rhmn^{Dd}(\,K_2 + mK_1) = \begin{cases} 7\left(\frac{m-1}{2}\right) + 2, if\ n\ is\ odd\ (\,m \geq 3) \\ 7\left(\frac{m}{2}\right) - 1, if\ n\ is\ even\ (m \geq 2) \end{cases}.$$

Proof:

Let $V(\,K_2 + mK_1) = \{v\} \cup \{u\} \cup \{u_i/i = 1,2,3 \ldots \ldots \ldots m\,\}$ and $E = \{u_i v,\ u_i u/i = 1,2 \ldots .m\}$..

So $diam^{\mathrm{Dd}}(\,K_2 + mK_1) = 2m + 4$

Without loss of generality, let $f(v_1) < f(v_2) < \cdots \ldots \ldots \ldots \ldots . < f(v_{m-1})$.

We shall check the radio heronian mean Dd-distance condition

$$D^{\mathrm{Dd}}(u,v) + \left\lceil \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rceil \geq 1 + diam^{\mathrm{Dd}}(G)$$

If m is odd, then

$$D^{Dd}(v_1,u_1) = D^{Dd}(v_1,u_2) = D^{Dd}(v_1,u_3) = m + 4, D^{Dd}(u_1,u_2) = n + 3, 1 \leq i \leq m$$

$$D^{Dd}(u_2,u_3) = m + 3\,, D^{Dd}(v_1,v_2) = m + 6, \qquad 1 \leq i \leq m$$

If (v_1,u_1) and (v_1,v_2) are adjacent,

$$D^{Dd}(v_1,u_1) + \left\lceil \frac{f(v_1) + \sqrt{f(v_1)f(u_1)} + f(u_1)}{3} \right\rceil \geq m + 4 + \left\lceil \frac{f(v_1) + \sqrt{f(v_1)f(u_1)} + f(u_1)}{3} \right\rceil \geq 2m + 5$$

$$\square^{\square\square}(\square_1,\square_2) + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_2)} + \square(\square_2)}{3} \right\rceil \geq \square + 6 + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_2)} + \square(\square_2)}{3} \right\rceil \geq 2\square + 5$$

If $(\square_1,\square_2)$ and $(\square_1,\square_2)$ are non adjacent,

$$\square^{\square\square}(\square_1,\square_2) + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_2)} + \square(\square_2)}{3} \right\rceil \geq \square + 3 + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_2)} + \square(\square_2)}{3} \right\rceil \geq 2\square + 5$$

Therefore $\square(\square_{\square}) = \square - \square - 1, 1 \leq \square \leq \square$

If $\square$ is even, then

$$\square^{\square\square}(\square_1,\square_1) = \square^{\square\square}(\square_1,\square_2) = \square^{\square\square}(\square_1,\square_3) = \square + 4, \square^{\square\square}(\square_1,\square_2) = \square + 4, 1 \leq \square \leq \square$$

$$\square^{\square\square}(\square_1,\square_2) = \square + 5, \qquad 1 \leq \square \leq \square$$

If $(\square_1,\square_1)$ and $(\square_1,\square_2)$ are adjacent,

$$\square^{\square\square}(\square_1,\square_1) + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_1)} + \square(\square_1)}{3} \right\rceil \geq \square + 4 + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_1)} + \square(\square_1)}{3} \right\rceil \geq 2\square + 5$$

$$\square^{\square\square}(\square_1,\square_2) + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_2)} + \square(\square_2)}{3} \right\rceil \geq \square + 5 + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_2)} + \square(\square_2)}{3} \right\rceil \geq 2\square + 5$$

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If $(□_1, □_2)$ and $(□_1, □_2)$ are non adjacent,

$$□^{□□}(□_1, □_2) + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_2)} + □(□_2)}{3} \right\rceil \geq □ + 4 + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_2)} + □(□_2)}{3} \right\rceil \geq 2□ + 5$$

Therefore $□(□_□) = □ - □, 1 \leq □ \leq □$

$$\text{Hence } □h□□^{□□}(□_2 + □□_1) = \begin{cases} 7\left(\frac{□-1}{2}\right) + 2, □□\ □\ □□\ □□□\ (□ \geq 3) \\ 7\left(\frac{□}{2}\right) - 1, □□\ □\ □□\ □□□□\ (□ \geq 2) \end{cases}.$$

Theorem: 2.3

The radio heronian mean Dd-distance number of a comb graph,

$$□h□□^{□}(□_□ \odot □_1) = 2□, \quad □ \geq 3$$

Proof:

Let $\{□_1, \ldots \ldots . . □_□\}$ and $\{□_1, □_2, \ldots \ldots . . □_□\}$ are the vertex set, where $\{□_1, □_2, \ldots \ldots . . □_□\}$ are the pendent vertices.

$□^{□□}(□_1, □_1) = □^{□□}(□_3, □_3) = □ + 1, □^{□□}(□_2, □_2) = □ + 2, □^{□□}(□_1, □_2) = □ + 2, 1 \leq □ \leq □$

$□^{□□}(□_1, □_2) = □ + 2\ , □^{□□}(□_1, □_2) = □ + 3, \quad 1 \leq □ \leq □$

So $□□□□^{Dd}(□_□ \odot □_1) = □ + 3$

Without loss of generality, let $□(□_1) < □(□_2) < \cdots \ldots \ldots \ldots \ldots . < □(□_{□-1})$.

We shall check the radio heronian mean Dd-distance condition

$$□^{Dd}(□, □) + \left\lceil \frac{□(□)+\sqrt{□(□)□(□)}+□(□)}{3} \right\rceil \geq 1 + □□□□^{Dd}(□)$$

If $(□_1, □_1)$ and $(□_1, □_2)$ are adjacent,

$$□^{□□}(□_1, □_1) + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_1)} + □(□_1)}{3} \right\rceil \geq □ + 1 + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_1)} + □(□_1)}{3} \right\rceil \geq 2□ + 4$$

$$□^{□□}(□_1, □_2) + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_2)} + □(□_2)}{3} \right\rceil \geq □ + 3 + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_2)} + □(□_2)}{3} \right\rceil \geq 2□ + 4$$

If $(□_1, □_2)$ are non adjacent,

$$□^{□□}(□_1, □_2) + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_2)} + □(□_2)}{3} \right\rceil \geq □ + 2 + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_2)} + □(□_2)}{3} \right\rceil \geq □ + 6$$

Therefore $□(□_□) = □ - □ - 1, 1 \leq □ \leq □$

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Hence $□h□□^{□}(□_{□} \odot □_{1}) = 2□,\ □ \geq 3$

Theorem: 2.4

The radio heronian mean Dd-distance number of a ladder graph,

$$□h□□^{□}(□_{□}) = \begin{cases} 7\left(\frac{□-1}{2}\right), □□\ □\ □□\ □□□\ (□ \geq 3) \\ 7\left(\frac{□}{2}\right) - 3, □□\ □\ □□\ □□□□\ (□ \geq 4) \end{cases}$$

Proof:

Let $\{□_{1}, \ldots \ldots . . □_{□}\}$ and $\{□_{1}, □_{2}, \ldots \ldots . . □_{□}\}$ are the vertex set, where $\{□_{1}, □_{2}, \ldots \ldots . . □_{□}\}$ are the pendent vertices.

So $□□□□^{Dd}(□_{□}) = 2□ + 3$

Without loss of generality, let $□(□_{1}) < □(□_{2}) < \cdots \ldots \ldots \ldots \ldots . < □(□_{□-1})$.

We shall check the radio heronian mean Dd-distance condition

$$□^{Dd}(□, □) + \left\lceil \frac{□(□) + \sqrt{□(□)□(□)} + □(□)}{3} \right\rceil \geq 1 + □□□□^{Dd}(□)$$

If □ is odd, then

$□^{□□}(□_{1}, □_{1}) = □^{□□}(□_{3}, □_{3}) = □ + 2, □^{□□}(□_{2}, □_{2}) = □ + 4,, □^{□□}(□_{1}, □_{2}) = □ + 4, 1 \leq □ \leq □$

$□^{□□}(□_{1}, □_{2}) = □ + 3\ , □^{□□}(□_{1}, □_{2}) = □ + 3, \quad 1 \leq □ \leq □$

If $(□_{1}, □_{1})$, $(□_{1}, □_{2})$ and $(□_{1}, □_{2})$ are adjacent,

$$□^{□□}(□_{1}, □_{1}) + \left\lceil \frac{□(□_{1}) + \sqrt{□(□_{1})□(□_{1})} + □(□_{1})}{3} \right\rceil \geq □ + 2 + \left\lceil \frac{□(□_{1}) + \sqrt{□(□_{1})□(□_{1})} + □(□_{1})}{3} \right\rceil \geq 2□ + 4$$

$$□^{□□}(□_{1}, □_{2}) + \left\lceil \frac{□(□_{1}) + \sqrt{□(□_{1})□(□_{2})} + □(□_{2})}{3} \right\rceil \geq □ + 3 + \left\lceil \frac{□(□_{1}) + \sqrt{□(□_{1})□(□_{2})} + □(□_{2})}{3} \right\rceil \geq 2□ + 4$$

$$□^{□□}(□_{1}, □_{2}) + \left\lceil \frac{□(□_{1}) + \sqrt{□(□_{1})□(□_{2})} + □(□_{2})}{3} \right\rceil \geq □ + 3 + \left\lceil \frac{□(□_{1}) + \sqrt{□(□_{1})□(□_{2})} + □(□_{2})}{3} \right\rceil \geq □ + 6$$

If $(□_{1}, □_{2})$ are non adjacent,

$$□^{□□}(□_{1}, □_{2}) + \left\lceil \frac{□(□_{1}) + \sqrt{□(□_{1})□(□_{2})} + □(□_{2})}{3} \right\rceil \geq □ + 4 + \left\lceil \frac{□(□_{1}) + \sqrt{□(□_{1})□(□_{2})} + □(□_{2})}{3} \right\rceil \geq □ + 6$$

Therefore $□(□_{□}) = 3(\frac{□-1}{2}) - □, 1 \leq □ \leq □$

If □ is even, then

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$\square^{\square\square}(\square_1, \square_1) = \square^{\square\square}(\square_4, \square_4) = \square + 1, \square^{\square\square}(\square_2, \square_2) = \square + 3, \square^{\square\square}(\square_1, \square_2) = \square + 3, 1 \leq \square \leq \square$

$\square^{\square\square}(\square_1, \square_2) = \square + 2\,, \square^{\square\square}(\square_1, \square_2) = \square + 2, \quad 1 \leq \square \leq \square$

If $(\square_1, \square_1)$,$(\square_1, \square_2)$ and $(\square_1, \square_2)$ are adjacent,

$$\square^{\square\square}(\square_1, \square_1) + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_1)} + \square(\square_1)}{3} \right\rceil \geq \square + 1 + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_1)} + \square(\square_1)}{3} \right\rceil \geq \square + 6$$

$$\square^{\square\square}(\square_1, \square_2) + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_2)} + \square(\square_2)}{3} \right\rceil \geq \square + 2 + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_2)} + \square(\square_2)}{3} \right\rceil \geq \square + 6$$

$$\square^{\square\square}(\square_1, \square_2) + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_2)} + \square(\square_2)}{3} \right\rceil \geq \square + 2 + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_2)} + \square(\square_2)}{3} \right\rceil \geq \square + 6$$

If $(\square_1, \square_2)$ are non adjacent,

$$\square^{\square\square}(\square_1, \square_2) + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_2)} + \square(\square_2)}{3} \right\rceil \geq \square + 2 + \left\lceil \frac{\square(\square_1) + \sqrt{\square(\square_1)\square(\square_2)} + \square(\square_2)}{3} \right\rceil \geq \square + 6$$

Therefore $\square(\square_\square) = 3\left(\frac{\square}{2}\right) - \square - 1, 1 \leq \square \leq \square$

Hence $\square h \square\square^{\square}(\square_\square) = \begin{cases} 7\left(\frac{\square-1}{2}\right), \square\square\ \square\ \square\square\ \square\square\square\ (\square \geq 3) \\ 7\left(\frac{\square}{2}\right) - 3, \square\square\ \square\ \square\square\ \square\square\square\square\ (\square \geq 4) \end{cases}$

Theorem: 2.5

The radio heronian mean Dd-distance number of a bistar,

$\square h \square\square^{\square\square}(\square_{\square,\square}) = \{3\square, \quad \square \geq 2.$

Proof:

Let $\square(\square_{\square,\square}) = \{\square\} \cup \{\square\} \cup \{\square_\square / \square = 1,2,3 \ldots \ldots \ldots \square\}$ and $\square = \{\square_\square\square, \ \square_\square\square, \square_\square\square_\square / \square = 1,2 \ldots . \square\}$.

Then , $\square^{\square\square}(\square_1, \square_3) = \square^{\square\square}(\square_1, \square_4) = \square^{\square\square}(\square_2, \square_5) = \square^{\square\square}(\square_2, \square_6) = \square + 4, \square^{\square\square}(\square_1, \square_2) = \square + 5, 1 \leq \square \leq \square$

So $\square\square\square\square^{Dd}(\square_{\square,\square}) = 2\square + 3$

Without loss of generality, let $\square(\square_1) < \square(\square_2) < \cdots \ldots \ldots \ldots \ldots . < \square(\square_{\square-1})$.

We shall check the radio heronian mean Dd-distance condition

$$\square^{Dd}(\square, \square) + \left\lceil \frac{\square(\square) + \sqrt{\square(\square)\square(\square)} + \square(\square)}{3} \right\rceil \geq 1 + \square\square\square\square^{Dd}(\square)$$

If $(\square_1, \square_2)$ and $(\square_1, \square_5)$ are adjacent,

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$$□^{□□}(□_1, □_2) + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_2)} + □(□_2)}{3} \right\rceil \geq □ + 5 + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_2)} + □(□_2)}{3} \right\rceil \geq 2□ + 4$$

$$□^{□□}(□_1, □_5) + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_5)} + □(□_5)}{3} \right\rceil □ + 3 + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_5)} + □(□_5)}{3} \right\rceil \geq 2□ + 4$$

If $(□_1, □_3)$ and $(□_2, □_5)$are non adjacent,

$$□^{□□}(□_1, □_3) + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_3)} + □(□_3)}{3} \right\rceil \geq □ + 4 + \left\lceil \frac{□(□_1) + \sqrt{□(□_1)□(□_2)} + □(□_2)}{3} \right\rceil \geq 2□ + 4$$

$$□^{□□}(□_2, □_5) + \left\lceil \frac{□(□_2) + \sqrt{□(□_2)□(□_5)} + □(□_5)}{3} \right\rceil □ + 3 + \left\lceil \frac{□(□_2) + \sqrt{□(□_2)□(□_5)} + □(□_5)}{3} \right\rceil \geq 2□ + 4$$

Therefore $□(□_□) = □ + □ - 2, 1 \leq □ \leq □$

Hence $□h□□^{□□}(□_{□,□}) = \{3□, \quad □ \geq 2.$

3. CONCLUSION

In this paper we studied on Radio Heronian mean Dd- distance of some operation graphs, which involves Dd- distance and Dd- diameter. We computed the Radio Heronian mean Dd-distance number by using some operation graphs and radio heronian mean number depends on the distance constraints.

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