THE TOTAL EDGE FIXING EDGE-TO-EDGE GEODETIC NUMBER OF A GRAPH

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ABSTRACT— In this article, we introduce the concept of the total edge fixing edge-to-edge geodetic number $g_{tefee}(G)$for an edge $e$of a graph G. The total edge fixing edge-to-edge geodetic *number* of certain classes of graphs including path, cycles, trees, complete graphs are studied. Also it is shown that for every pair of positive integers with $2 \leq a \leq b$, there exists a graph G such that $g_{tee}(G) = a$ and $g_{tefee}(G) = b$,for some edge $e \in E(G)$.

Keywords—total edge fixing edge-to-edge geodetic number,total edge-to-edge geodetic number, edge-to-edge geodetic number, distance ,edge-to-edge distance

I.INTRODUCTION

By a *graph* $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The *order* and *size* of G are denoted by p and q respectively. We consider connected graphs with at least three vertices. For basic definitions and terminologies we refer to [1].

The following theorems are used in sequel.

Theorem 1.1. [1] If v is an extreme vertex of a connected graph G, then every edge-to-edge geodetic set contains at least one extreme edge is incident with v.

Theorem 1.2. [1] For any non-trivial tree T with $k$end vertices,$g_{ee}(T) = k$.

II THE TOTAL EDGE FIXING EDGE-TO-EDGE GEODETIC

Definition: 2.1. Let $e$be an edge of a graph$G$. A set $M(e) \subseteq E(G) - \{e\}$ is called a *total edge fixing edge-to-edge geodetic set* of e of a graph G, if the sub graph induced by $M(e), G[M(e)]$has no isolated edges. The minimum cardinality of a total edge fixing edge-to-edge geodetic set is called *total edge fixing edge-to-edge geodetic number* of $G$and is denoted by $g_{tefee}(G)$. Any total edge fixing edge-to-edge geodetic set of cardinality $g_{tefee}(G)$is ag_{tefee}-set of G.

Example: 2.2. For the graph$G$ given in Figure 2.1, the total edge fixing edge-to-edge geodetic sets of

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each edge of G is given in the following Table I

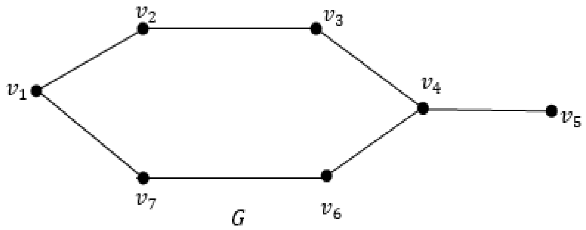

Figure 2.1

Table: I

Fixing Edge (e)	**Minimum total edge fixing edge-to-edge geodetic sets ($M(e)$)**	$g_{tefee}(G)$
v_1v_2	$\{v_4v_5, v_4v_6\}$	2
v_2v_3	$\{v_4v_5, v_4v_6, v_6v_7\}$	3
v_3v_4	$\{v_1v_2, v_1v_7, v_4v_6, v_4v_5\}$, $\{v_1v_7, v_6v_7, v_4v_6, v_4v_5\}$, $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$, $\{v_1v_2, v_1v_7, v_3v_4, v_4v_5\}$	4
v_4v_5	$\{v_1v_2, v_1v_7\}$	2
v_1v_7	$\{v_3v_4, v_4v_5\}$	2
v_6v_7	$\{v_2v_3, v_3v_4, v_4v_5\}$	3
v_4v_6	$\{v_1v_2, v_1v_7, v_3v_4, v_4v_5\}$, $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$, $\{v_1v_2, v_1v_7, v_4v_6, v_4v_5\}$, $\{v_1v_7, v_6v_7, v_4v_6, v_4v_5\}$	4

Theorem: 2.3. For the graph $G = C_p (p \geq 4)$, $g_{tefee}(G) = 2$, for any edge e of $E(G)$.

Proof: Let $C_p: v_1, v_2, v_3, \ldots, v_p$ be the cycle and$e$be an edge of C_p. We have the following two cases.

Case(i). Let$p$be even. Let f be the antipodal edge of $e$of$G$ and h be any edge adjacent to f.Then $\{f, h\}$ is a total edge fixing edge-to-edge geodetic set of e of G so that$g_{tefee}(G) = 2$.

Case(ii). Let$p$beodd. Let g and f be the antipodal edges of e of G.Then $\{g, f\}$is a total edge fixing edge-to-edge geodetic set of e of G so that $g_{tefee}(G) = 2$.

Theorem: 2.4. For the complete graph $G = K_p (p \geq 4)$ with vertex set $\{v_1, v_2, v_3, \ldots, v_p\}$, $g_{tefee}(K_p) = p - 1$for any edge $e \in \{v_1v_2, v_2v_3, \ldots, v_{p-1}v_p\}$.

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Proof: Let e be an edge of K_p. Without loss of generality, let $e = v_1v_2$.

Case$(\boldsymbol{i})p = 4$. Then $S = \{v_2v_3, v_4v_5\}$ is a g_{tefee}-set of G so that $g_{tefee}(G) = 2$.

Case$(\boldsymbol{ii})p \geq 5$. Then $S_1 = \{v_3v_4, v_4v_5, \dots, v_{p-1}v_p\}$ is a g_{tefee}-set of G so that $g_{tefee}(G) \leq p - 3$. We prove that $g_{tefee}(G) = p - 3$. On the contrary suppose that $g_{tefee}(G) \leq p - 4$. Then there exists a g_{tefee}-set S' of e of Gsuch that $|S'| \leq p - 4$. Since $G[S']$ is a path, there exists $f \in E(G)$ such that f do not lie on a geodesic joining a pair of edges of S', which is a contradiction. Therefore $g_{tefee}(G) = p - 3$.

Theorem: 2.5. For the complete bipartite graph $= K_{m,n}(2 \leq m \leq n), g_{tefee}(K_p) = n + m - 2$, for any edge $e\ of\ E(G)$.

Proof: Let $U = \{u_1, u_2, \dots, u_m\}$and $V = \{v_1, v_2, \dots,$
$v_n\}$ be a bipartition of G. Let us fix the edge $e = u_1v_1$ in G. Let $S = \{u_2v_2, u_3v_3, \dots, u_mv_m, u_mv_{m+1}, \dots, u_m, v_n, v_1u_2,$
$v_2u_3, \dots, v_{m-1}u_m\}$. Clearly S is a total edge fixing edge-to-edge geodetic set of $e = u_1v_1$ of G so that $g_{tefee}(G) \leq n + m - 2$. On the other hand, let $g_{tefee}(G) \leq n + m - 3$.Then the total edge fixing edge-to-edge geodetic set S' of the edge e exits such that $|S'| \leq n + m - 3$.Consequently there is an edge, say $e \in S$ such that $e \notin S'$ and e is not incident with any vertex set of $V(S')$. Therefore e does not lie on a geodesic joining a pair of edges of S'. Hence S' is not a total edge fixing edge-to-edge geodetic set of G, which is a contradiction. Hence $g_{tefee}(K_p) = n + m - 2$.

III. CONCLUSION

With the contribution of the total edge fixing edge-to-edge geodetic number of a graph, we can introduce the forcing total edge fixing edge-to-edge geodetic number$f_{g_{tefee}}(G)$ of an edge e of G.The forcing total edge fixing edge-to-edge geodetic *number* of certain graphs can be studied.

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