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SOME NON LINEAR REGRESSION MODELS

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Abstract

Nonlinear regression is a form of regression analysis in which data is fit to a model and then expressed as a mathematical function. Simple linear regression relates two variables (X and Y) with a straight line (y = mx + b), while nonlinear regression relates the two variables in a nonlinear (curved) relationship. The goal of the model is to make the sum of the squares as small as possible. The sum of squares is a measure that tracks how far the Y observations vary from the nonlinear (curved) function that is used to predict Y. It is computed by first finding the difference between the fitted nonlinear function and every Y point of data in the set. Then, each of those differences is squared. Lastly, all of the squared figures are added together. The smaller the sum of these squared figures, the better the function fits the data points in the set.

Key Words: COBB-DOUGLAS functions, Non linear Regression Models, Least Squares estimation, Linear Approximation

INTRODUCTION

The majority of economic theory is focused with the interrelationships between variables. When expressed in mathematically, these relationships can be forecast the impact of one variable on another. For example, a consumer’s quantity demanded (C) is a function of the commodity’s price (P). This can be written as q = f(P). Similarly, the supply function can be expressed as S = f(P) and the cost function can be written as C = f(q). In the specific form, these functional connections define the

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dependency of the dependent variable on the independent variable(S). The functional form can be linear, quadratic, logarithmic, exponential, hyperbolic, and so on.

We shall took at a simple linear regression model, where the relationship between two variables are expressed as a linear function, multiple regression models, and non-linear regression models, as well as estimation and testing procedures, in this paper. We shall also look at some non-linear models that have linear parameters but not necessarily linear variable.

NON-LINEAR REGRESSION MODELS

In terms of the variables, these models are non-linear, but linear in terms of the parameters. The key feature of such models is that they may be transformed into regular linear models using appropriate variable transformations. The first order criteria for least squares parameter estimation in non-linear regression model are non-linear functions of the parameters. The following sections discuss several nonlinear models.

A non-linear regression model is one with a non-linear regression function in the unknown coefficient vector β (Say). Consider the following non-linear regression model as

$$Y_t = \beta_1 e^{\beta_2 X_t} + U_t$$

If ϵ_t were multiplicative rather than additive one could get a linear model by taking logarithm. Applying the method of least squares, we minimize

$$\frac{1}{2}\sum_t \left(Y_t - \beta_1 e^{\beta_2 X_t}\right)^2$$

The normal equations to obtains β_1 and β_2 are

$$\sum_t (Y_t - \beta_1 e^{\beta_2 X_t}) e^{\beta_2 X_t} = \sum_t Y_t e^{\beta_2 X_t} - \beta_1 \sum_t e^{2\beta_2 X_t} = 0$$

$$\sum_t (Y_t - \beta_1 e^{\beta_2 X_t}) e^{\beta_2 X_t}.X_t = \sum_t Y_t X_t e^{\beta_2 X_t} - \beta_1 \sum_t X_t e^{2\beta_2 X_t} = 0$$

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These are called non-linear equations in β_1 and β_2. In this case the regression function $\beta_1 e^{\beta_2 X_t}$ is non-linear in the parameters β_1 and β_2. As a result the normal equations are also non-linear functions. In Econometrics, there kinds of non-linear models have been found . The first is a universal non-linear regression model with an additive disturbance as

$$Y_t = g(X_t,\theta)+U_t; \quad t=1,2,\ldots\ldots\ldots,n$$

Where, X_t is a k-component vector denoting t^{th} observation on k- explanatory variables.

θ is a k - component vector of parameters

g is a non-linear function and the residual U_t iid with mean zero and variance σ^2.

The second is a G- variate non-linear regression model with additive disturbances. The i^{th} regression equation is

$$Y_{ti} = g_i\,(X_t,\theta)+U_{ti}\,; \quad t=1,2,\ldots..,n \qquad i=1,2,\ldots.,G$$

There are G dependent variables $Y_{t1}, Y_{t2}, \ldots\ldots, Y_{tG}$. The i^{th} regression function g_i may not include all explanatory variables in X_t and all parameters in θ as arguments, the model can be written as

$$Y_t = g(X_t,\theta)+ U_t$$

Where Y_t is a vector of G-dependent variables

g is a vector of G-regression functions

U_t is assumed to be iid, having zero mean and covariance matrix Ω.

The third model is a system of non-linear simultaneous equations with additive disturbances. The i^{th} structural equation is

$$\Phi_i(Y_{ti}, X_t, \theta_i) = U_{ti}, \quad i= 1,2,\ldots..,G$$

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Here the i[th] endogenous variable $Y_{ti}$is an explioit function of other endogenous variables in Y_t and pre-determined variables X_t; θ_i is a vector of parameters in the above equation.

Now we can write the model as

$$\Phi(Y_t, X_t, \theta) = U_t$$

The residual vector U_t is assumed to be iid, having mean zero and covariance matrix Σ. The equation is a generalization of non-linear multiple regression model, the equation is a generalization of system of linear regression equations and the equation is a generalization of a system of linear simultaneous equations.

NON LINEAR REGRESSION MODEL: PARAMETERS ARE IN NON-LINEAR FORM

Consider a non-linear relationship of the form

$$Y= \beta_0 X_1^{\beta_1} X_2^{\beta_2}$$

Taking Natural logarithms on both sides

$$\ln Y = \ln \beta_0 + \beta_1 \ln X_1 + \beta_2 \ln X_2$$

Which is again linear in the form

$$\tilde{Y} = \tilde{\beta}_0 + \beta_1 \tilde{X}_1 + \beta_2 \tilde{X}_2$$

Where $\tilde{Y} = \ln Y$

$\tilde{\beta}_0 = \ln \beta_0$

$\tilde{X}_1 = \ln X_1$

$\tilde{X}_2 = \ln X_2$

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A unique assumption concerning the disturbance component for the original model is required if the classical linear regression model is to be acceptable for the above log-linear equation. There are two ways to introduce the disturbance term.

(i) The original model with multiplicative disturbance term can be written as

$$Y = \beta_0 X_1^{\beta_1} X_2^{\beta_2} U$$

Here, U is called multiplicative error, then

Taking Natural logarithms on both sides

$$\ln Y = \ln \beta_0 + \beta_1 \ln X_1 + \beta_2 \ln X_2 + \ln U$$

Which is non-linear Regression model but it is in linear form is as follows

$$\tilde{Y} = \tilde{\beta}_0 + \beta_1 \tilde{X}_1 + \beta_2 \tilde{X}_2 + \tilde{U}$$

(ii) Secondly, the original model with additive disturbance term can be written as

$$Y = \beta_0 X_1^{\beta_1} X_2^{\beta_2} + U$$

Here U is called additive error. In this case, no transformation of variables could lead to a regression equation that would be linear in β's. So the equation would have to be classified as an intrinsically non-linear.

ESTIMATION OF PARAMETERS IN NON-LINEAR REGRESSION MODELS

THEORY OF LEAST SQUARES ESTIMATION IN NON – LINEAR MODELS

$$Y = f(X_1, X_2, \ldots X_K;\ \beta_1, \beta_2, \ldots \beta_P) + U_i \quad i=1,2, \ldots, m$$

Where Y is the dependent variable

X_i's are the independent variables

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$\beta_1, \beta_2, \ldots \ldots \beta_p$ are 'p' parameters

U is error variable

F is non-linear function form which may be known and n is the number of observations on each variable

$$Y^* = \begin{bmatrix} Y_1 \\ Y_2 \\ . \\ . \\ . \\ . \\ . \\ Y_m \end{bmatrix}_{m\times 1}, \quad X^* = \begin{bmatrix} X_{11} & X_{12} & \ldots\ldots & X_{1k} \\ X_{21} & X_{22} & \ldots\ldots & X_{2k} \\ . & . & \ldots\ldots & . \\ . & . & \ldots\ldots & . \\ . & . & \ldots\ldots & . \\ . & . & \ldots\ldots & . \\ . & . & \ldots\ldots & . \\ X_{m1} & X_{m2} & \ldots\ldots & X_{mk} \end{bmatrix}_{m\times k} \quad \beta^* = \begin{bmatrix} \beta_1 \\ \beta_2 \\ . \\ . \\ . \\ . \\ . \\ \beta_p \end{bmatrix}_{p\times 1} \quad U^* = \begin{bmatrix} U_1 \\ U_2 \\ . \\ . \\ . \\ . \\ . \\ U_m \end{bmatrix}_{m\times 1}$$

And $f(X_{ji}, \beta) = \begin{bmatrix} f(X_{j1}, \beta) \\ f(X_{j2}, \beta) \\ . \\ . \\ . \\ f(X_{jm}, \beta) \end{bmatrix}$ $\quad j=1,2,\ldots\ldots, k$

Now the above non-linear model can be written in matrix notation as

$$Y = f(X, \beta) + U$$

Where Y is (m×1), X is (m × k), β is (p×1) and U is (m×1) matrices. Assume that the error in are iid's with

$$U \sim N(0, \sigma^2 I)$$

But the exact form of the distribution is unknown

Now define the residual sum of squares, as

$$R(\tilde{\beta}) = [Y - f(X_i, \tilde{\beta})]^{'} [Y - f(X_i, \tilde{\beta})]$$

The non-linear least squares estimator $\tilde{\beta}$ can be obtained by minimizing $R(\tilde{\beta})$ with respect to $\tilde{\beta}$ and then solve the non-linear normal equations for $\tilde{\beta}$.

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Consider the residual sum of squares

$$R(\tilde{\beta}) = \Sigma[Y_i - f(X_{i,}\tilde{\beta})]^2$$

Here, X_i is a (1×k) row vector of k independent variables. The p non-linear normal equations are given by

$$\frac{\partial R(\tilde{\beta})}{\partial \tilde{\beta}} = 0 \Rightarrow -2\,\Sigma[Y_i - f(X_{i,}\tilde{\beta})]\left[\frac{\partial f(X_{i,}\tilde{\beta})}{\partial \tilde{\beta}_j}\right] = \quad \forall\ 0\ j=1,2,\ .\ .\ .\ .\ .\ ,\ k$$

Since, $f(X_{i,}\tilde{\beta})$ is a non-linear in β's the normal equation will be non-linear in X's and β's.

METHOD OF LINEAR APPROXIMATION

Consider a non-linear regression model in matrix notation as

$$Y = f(X_{i,}\beta) + U$$

Assume iid $U \sim N(0, \sigma^2 I)$

Now consider the approximation fraction function as $f(X_i, \beta)$ at an initial point β^*. Using Taylor's series expansion and by neglecting second order derivatives. We have

$$f(X_i, \beta) \simeq f(X, \beta^*) + Z(\beta - \beta^*) + \left[\frac{\partial f(X_{i,}\beta)}{\partial \beta}\right]_{\beta=\beta^*}[\beta - \beta^*]$$

$$\simeq f(X, \beta^*) + Z(\beta - \beta^*)$$

Where $Z = \left[\frac{\partial f(X_{i,}\beta)}{\partial \beta}\right]_{\beta=\beta^*}$

Now (5.13.6) becomes,

$$Y = f(X, \beta^*) + Z(\beta - \beta^*) + U$$

$$Y^* = Z\beta + U$$

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Where $Y^* = Y - f(X, \beta^*) + Z\beta^*$. Using OLS estimation procedure to we get a second round estimate for β which is given by

$$\beta^{**} = (Z^{l}Z)^{-l}Z^{l}(Z\beta + U)$$

$$\beta^{**} = (Z^{l}Z)^{-l}(Z^{l}Z)\beta + (Z^{l}Z)^{-l}Z^{l}U$$

$$\beta^{**} = \beta + (Z^{l}Z)^{-l}Z^{l}U$$

Continue this process when the process has conversed to $\tilde{\beta}^*_{n-1} = \tilde{\beta}^*_n$ and

$V(\beta^{**}) = \sigma^2(Z^{l}Z)^{-l}$ and also we have $\hat{\sigma}^2 = \frac{p^{l}p}{n-p}$.

COBB-DOUGLAS TYPE FUNCTIONS ESTIMATION WITH MULTIPLICATIVE AND ADDITIVE ERRORS

The estimation of function of the type

$$Y = \beta_0 X_1^{\beta_1} X_2^{\beta_2} \ldots\ldots X_K^{\beta_K}$$

Occurs in economics in various contexts such as in the study of demand functions and production functions. The stochastic term in models of type is either specified to be additive of type

$$Y = \beta_0 X_1^{\beta_1} X_2^{\beta_2} \ldots\ldots X_K^{\beta_K} + U$$

Or, much more frequently to be multiplicative as

$$Y = \beta_0 X_1^{\beta_1} X_2^{\beta_2} \ldots\ldots X_K^{\beta_K} . U$$

In either case it is generally assumed that U is distributed according to $N(0, \sigma^2 I)$. The difference between the two types of specifications of the error terms are serial.

(i) The conditional expectation of the dependent variable in is

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$$E(Y/X) = \beta_0 X_1^{\beta_1} X_2^{\beta_2} \ldots\ldots X_K^{\beta_K}$$

Where as in it is

$$E(Y/X) = \beta_0 X_1^{\beta_1} X_2^{\beta_2} \ldots\ldots X_K^{\beta_K} e^{\sigma^2}$$

(ii) The model given by is homoscedastic, where as is heteroscedastic and the conditional variance of Y being

$$V(Y/X) = [E(Y/X)]^2 \left(e^{\sigma^2} - 1\right) = [E(Y/X)^2] - [E(Y/X)]^2$$

The estimation is straight forward. The parameters of can be estimated with non-linear techniques by minimizing.

$$\sum_t \left(Y_t - \beta_0 X_{1t}^{\beta_1} X_{2t}^{\beta_2} \ldots\ldots X_{Kt}^{\beta_K}\right)^2$$

With respect to the parameters $\beta_0, \beta_1, \beta_2, \ldots\ldots \beta_K$. The resulting estimates are maximum likelihood estimates. The parameters of are estimate by taking logarithms on both sides and computing the resulting linear regression.

Summary and Conclusion :

Specified various criteria for the selection of non linear regression models besides some advanced general criteria for Regression analysis.

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