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FUZZY LINEAR PARTIAL DIFFERENTIAL ALGEBRAIC EQUATIONS USING SIMULINK

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ABSTRACT: This work uses Simulink to derive optimum control for Fuzzy linear Partial Differential Algebraic Equations (FPDAE) with quadratic performance. The FPDAE is converted into a Fuzzy Differential Algebraic Equation (FDAE) by applying the technique of lines. Therefore, determining the optimal control of the associated FDAE will result in the optimal control of FPDAE. The objective is to use Simulink's Matrix Riccati Differential Equation (MRDE) solutions to achieve optimum control with less mathematics work. The Simulink approach to the problem provides a solution with more accuracy on a qualitative level. Hence, this model can evaluate the solution at any desired number of points spending negligible computing time and memory and the solution curves can be obtained from the model without writing any code. Hence, this model will achieves better results interms of accuracy and computing time.

KEYWORDS: Fuzzy Differential Algebraic Equation (FDAE), Fuzzy linear Partial Differential Algebraic Equation (FPDAE), Matrix Riccati Differential Equation (MRDE), Optimal Control and Simulink.

I. INTRODUCTION

Optimal control theory is considered as a modern extension of the classical calculus of variations; however, it differs from calculus of variations in that it uses control variables to optimize the function. The development of the mathematical theory for optimal control began in the early 1950's, partially in response to problems in various branches of engineering and economics. The study of classical optimal control theory from different viewpoints greatly attracted the attention of many mathematicians, and the detailed arguments can be found in many textbooks, for instance, [1], and references therein.

Moreover, optimal control strategy, i.e., solving necessary conditions for optimality, can be applied in several fields, such as economy, biology, and process engineering.

Linguistic IF-THEN rules with fuzzy antecedent and consequent elements make up a fuzzy system. The mapping from the input space to the output space is static and nonlinear. Rather than being hazy sets, the inputs and outputs are distinct actual values. The inference mechanism employs the fuzzy rules in the rule base to form fuzzy conclusions or fuzzy aggregations, and the defuzzification block then converts these fuzzy conclusions back into crisp outputs. Initially, the crisp inputs are converted to fuzzy sets by the fuzzification block. The conventional fuzzy system consists of a singleton fuzzifier, product inference engine, center average defuzzifier, and Gaussian membership functions [2].

Fuzzy systems offer two primary benefits for control and modeling applications: (i) the ability to handle uncertain or approximation reasoning, which may be particularly challenging to represent mathematically; and (ii) the ability to handle decision-making issues involving estimated values under incomplete or uncertain information [3]. For any system, stability and optimality are the most crucial prerequisites. It appears that the field of optimum fuzzy control is completely unbounded in terms of optimality. In certain problems, control variables enter the Hamiltonian linearly, either via the objective function or the

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dynamic system or both. This type of problem is called Linear Optimal Control.

There are both differential and algebraic equations in singular systems. That is to say, the differential part's solution is constrained by the algebraic equations[4]. These systems are sometimes referred to as generalized state-space, degenerate, descriptor, or semi-state systems. In many applications, including electrical networks, aircraft dynamics, neutral delay systems, chemical, thermal, and diffusion processes, large-scale systems, robotics, biology, etc., the system naturally develops as a linear approximation of system models or linear system models.

Recently, a lot of works done in the field of the fuzzy optimal control problem have only examined problems with one control and one dependent state variable; however, many times, we will wish to examine fuzzy optimal control problems which arise in a wide variety of scientific and engineering applications such as physics, chemical engineering, and economy, with more variables (more controls and more states). It seems that it is a good idea to consider fuzzy optimal control problems of several variables and discuss how to handle such problems [5]. Further, treating a special type of fuzzy optimal control problems such as problems having a type of constraint known as an isoperimetric constraint and problems involving higher order differential equations has been presented.

Simulink is a MATLAB add-on package that many professional engineers use to model dynamical processes in control systems. Simulink allows to create a block diagram representation of a system and run simulations very easily. Simulink is really translating a block diagram into a system of ordinary differential equations. Simulink is the tool of choice for control system design, Digital Signal Processing (DSP) design, communication system design and other simulation applications. This paper focuses upon the implementation of Simulink approach for solving MRDE in order to get the optimal control of the PDAE.

It shows that the optimal feedback control and the minimum cost are characterized by the solution of a Riccati equation [6]. Solving the Matrix Riccati Differential Equation (MRDE) is a central issue in optimal control theory. The needs for solving such equations often arise in analysis and synthesis such as linear quadratic optimal control systems, robust control systems with H**2** and H –control performance criteria, stochastic filtering and control systems, model reduction, differential games etc. One of the most intensely studied nonlinear matrix equations arising in Mathematics and Engineering is the Riccati equation. This equation, in one form or another, has an important role in optimal control problems, multivariable and large scale systems, scattering theory, estimation, detection, transportation and radiative transfer. The solution of this equation is difficult to obtain from two points of view. One is nonlinear and the other is in matrix form. Most general methods to solve MRDE with a terminal boundary condition are obtained on transforming MRDE into an equivalent linear differential Hamiltonian system. By using this approach, the solution of MRDE is obtained by partitioning the transition matrix of the associated Hamiltonian system. Another class of methods is based on transforming MRDE into a linear matrix differential equation and then solving MRDE analytically or computationally.

II. LITERATURE SURVEY

M. Valipour and Y. Wang., et.al [7] Concept algebra is a denotational mathematics for rigorously manipulating formal concepts and their algebraic operations in knowledge representation,

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semantic analyses, and machine learning. Properties of concept algebra are formally studied in order to elaborate the nature of formal concepts and their algebraic operations. This leads to a set of algebraic rules in the categories of relational, reproductive, and compositional operations on formal concepts. Relationship between algebraic operations of concept algebra is rigorously described. Proofs are provided for the algebraic rules of concept algebra elaborated by examples. This work enables a rigorous implementation of concept algebra in cognitive knowledge base manipulations and cognitive machine learning.

C. M. R. Caridade., et.al [8] Linear Algebra is one of the main mathematical fields, which is used in many technical subjects. Inability to realize the importance of Linear Algebra and its applications in technical subjects is one of the reasons of poor academic performance of students. The typical teaching approach separates the Linear Algebra's from technical subject. This does not necessarily promote student's awareness in both subjects. This paper presents a new teaching approach which combine Linear Algebra and Image Processing contents. The methodology presented is based in teaching Linear Algebra contents such as matrices and matrix transformation using Image Processing applications. This could lead to a whole new interesting learning environment, new teaching approaches that are more stimulating for students and more productive and dynamic for teachers.

C. Bo, Z. Xingyou, Z. Pengfei, C. Cong, L. Wenxue and Z. Kang., et.al [9] sound and complete Gentzen-typed deduction system is given for a Boolean algebra logic, where the logical connective - is missed and an assignment is a function from the propositional variables to a Boolean algebra. A correspondence between the quotient of formulas under the valid equivalences of formulas and lattices is given, corresponding to the correspondence between the quotient of formulas under the classical valid equivalences of formulas and Boolean algebras.

H. Geng, X. Shi, X. Yin, Z. Wang and H. Zhang., et.al [10] diversity of QoS (Quality-of-Service) requirements of Internet applications motivates various QoS routing algorithms that take different QoS metrics into consideration. Routing algebra has been proposed as a framework to study the fundamental properties of QoS routing algorithms, such as their optimality and loop-freeness. However, for multipath QoS routing, little has been done in these aspects. Existing multipath QoS routing algorithms often take a rather conservative approach to guarantee loop-freeness, at the cost of efficiency. On the other hand, simply adapting existing efficient multipath routing algorithms to support various QoS metrics cannot guarantee correctness. In face of that, we propose a routing metric algebra for multipath QoS routing in link state networks, where a key property of the routing metrics called isotonicity, which plays an important role. To let routers efficiently and correctly find multiple next-hops for each destination, we also develop two distributed multipath QoS routing algorithms. The algorithms are run locally and independently, without exchanging messages other than the basic link states. They are specifically tailored for algebras with strict or non strict isotonicity, and their correctness are formally proved.

R. Kaur and Pooja, et.al [11] CAPTCHA is widely research field become a ubiquitous defense to protect open web resources from automated access. The objective of CAPTCHA is to give genuine services to every user by minimize automated approaches. An efficient CAPTCHA must be not only human friendly, but also robust enough to resist to machine attacks for pass CAPTCHA test. Many CAPTCHAs

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have been proposed in literature like text-graphical based, video based, mathematical or questions based. The miscreants attempt to effectively bypass these CAPTCHA test by use optical recognition or machine learning techniques. The design and implementation of CAPTCHA lie in the domain of artificial intelligence. We aim to design a CAPTCHA, which will effectively use by modern digitize world and turn to reduce the optical recognition and other automated attacks. In this paper, for better security issues, a non OCR CAPTCHA design called digital CAPTCHA based on Mathematical Boolean algebra by use digital logic gates. The different logic gates design which depict shapes like (AND, OR, NOT, NAND, NOR). Each gate executes multiple answers based on Boolean algebra by use Boolean laws. For prove human, user recognize shapes of logic gates and choose a correct answer from multiple answers. The experimental results analyze by multiple technical and non technical users with different age groups. The resulting values show that our proposed system has high probability and security and resists the visual attacks with existing math CAPTCHAs.

B. Erabadda, S. Ranathunga and G. Dias, et.al [12] presents a system that automatically identifies errors made by students in answering algebra questions that require multiple steps. The types of algebra questions we consider include linear equations with fractions and quadratic equations. We have already developed a system that is capable of grading multi-step answers to the aforementioned two types of questions and awarding full/partial credit according to a marking scheme. The error identification module works on top of this previous system. It was evaluated using data from two sources: government schools and a tuition class in Sri Lanka. The mistakes identified by the system were compared against feedback by two independent teachers. The results showed that the system identified the student mistakes with more than 85% accuracy for both types of questions.

V. Skala, M. Smolik and M. Martynova, et.al [13] describes a new approach to a solution of multidimensional dynamical systems using the La-place transform and geometrical product, i.e. using inner product (dot product, scalar product) and outer product (extended cross-product). It leads to a linear system of equations Ax=0 or Ax=b which is equivalent to the outer product if the projective extension of the Euclidean system and the principle of duality are used. The paper explores property of the geometrical product in the frame of multidimensional dynamical systems. The proposed approach enables to avoid division operation and extents numerical precision as well. It also offers applications of matrix-vector and vector-vector operations in symbolic manipulation, which can lead to new algorithms and/or new formula. The proposed approach can be applied also for stability evaluation of dynamical systems. In the case of numerical computation, it supports vector operation and SSE instructions or GPU can be used efficiently.

M. Cartagena, R. Pillajo, O. Camacho, A. Rosales and G. Scaglia,et.al [14] proposes a controller based on Numerical Methods and Linear Algebra (NMLA). It is applied to systems with inverse response that are approximated to reduced second order models. The design of the controller is presented to a non-linear and one linear systems, to make a comparison of the results between a control using the complete model of the system and one using the reduced.

A. T. Rahem, M. Ismail, N. F. Abdullah and I. A. Najm, et.al [15] graph theory is how to find the shortest paths between two

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vertices, thus covering a wide range of research areas. So far, almost all previous studies have used Algorithms to find such path. However, this consumes more or large search spaces, and it tends to be more complex. Therefore, this paper presents a new mathematical method to finding the shortest path in an undirected graph. The method involves only Boolean algebra to reduce the search space and computational complexity. This method is simple and even faster than linear programming.

III. METHODOLOGY

Simulink is an interactive tool for modelling, simulating and analyzing dynamic systems. It enables engineers to build graphical block diagrams, evaluate system performance and refine their designs. Simulink integrates seamlessly with MATLAB and is tightly integrated with state flow for modelling event driven behavior. Simulink is built on top of MATLAB. A Simulink model for the given problem can be constructed using building blocks from the Simulink library. The solution curves can be obtained from the model without writing any code.

A Simulink model is constructed for the following system of two differential equations as shown in Fig. 1.

$$x(t) = -x(t) + 1, x(0) = -1 \qquad --1$$

$$y(t) = -y(t) + 1, y(0) = 1 \qquad --2$$

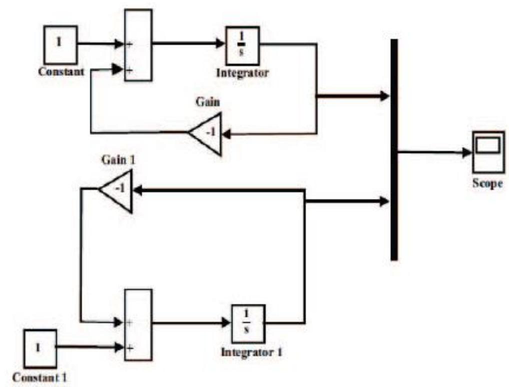

Fig.1: Circuit Diagram

As soon as the model is constructed, the Simulink parameters can be changed according to the problem. The solution of the system of differential equation can be obtained in the display block by running the model.

Procedure for Simulink Solution

Step 1. Select the required number of blocks from the Simulink Library.
Step 2. Connect the appropriate blocks.
Step 3. Make the required changes in the simulation parameters.
Step 4. Run the Simulink model to obtain the solution.

IV. RESULT ANALYSIS

In this section result analysis for fuzzy liner partial differential equations for optimal control is observed.

Table.1: Performance Analysis

Parmeters	FDAE	Linear Equations
Computing Time	6123	9584
Accuracy	99	86

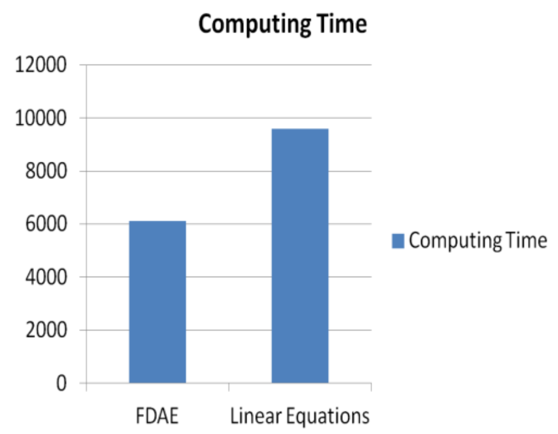

Fig.2: Computing Time Comparison Graph

In Fig.2 computing comparision graph is seen between FDAE and linear equation.

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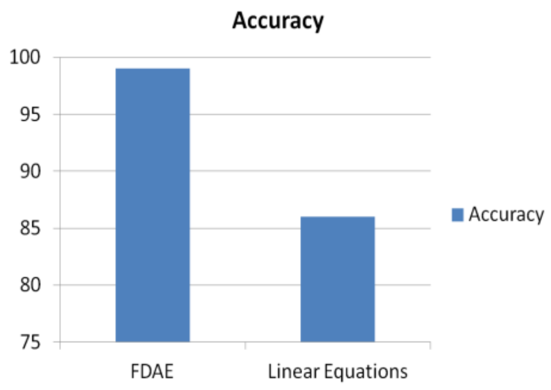

Fig.3: Accuracy Comparison Graph

In Fig.3 accuracy comparison graph is seen between FDAE and linear equation.

V. CONCLUSION

The optimal control of FPDAE has been obtained by finding the optimal control of the corresponding FDAE. The optimal control of FDAE is found out by solving the relative MRDE. The numerical results of the MRDE in that the Simulink solutions are much more efficient and accurate. The long calculus time of the MRDE is avoided by using a Simulink. The solution curves can be obtained from the model without writing any code. The efficient optimal solution is done with PC, CPU 2.0 GHz in MATLAB. Solution of MRDE for FPDAE is obtained without writing any code. Every researcher irrespective of subjects can solve algebraic and differential equations without writing any code and also knowing the Mathematical procedures or techniques. Hence, this model achieves better results interms of accuracy and reducing computing time.

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