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Research Paper **UGC CARE Listed (Group -I) Journal**

ON PROPER COLORING OF CORONA GRAPH, FAN GRAPH, AND BUTTERFLY GRAPH

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Abstract:-

In this paper we prove that the chromatic number of corona graph, $C_n\Theta k_{1,3}$ is 2 or 3, We also show that chromatic number of the two copies of Fan graph Fn by a path is 3, Further, we prove that the butterfly graph $B_{n,m}$ which admits a proper coloring and its chromatic number is 3.

Keywords:- Chromatic number, corona $C_n\Theta k_{1,3}$ Fan graph, Butterfly graph.

Introduction:-

We begin with simple, finite, connected and undirected graph G=(V(G),E(G)) with p vertices and q edges. For standard terminology and notations we follow Ferdin and Mobius [2]. We will provide brief summary of definitions and other information which are necessary for the present investigators.

Definition 1:

Graph coloring is a special case of Graph Labeling. It is an assignment of labels traditionally called colors to elements of a graph subject to certain constraints. In its simplest form it is the way of coloring the vertices of a graph such that no two adjacent vertices share the same color.

Definition 2:

The chromatic number of a graph is the smallest number of colors needed to color the vertices of a graph so that no two adjacent vertices share the same color. i.e. chromatic number is the smallest possible value to obtain a k- coloring and is denoted by $\Psi(G)$[3]

Definition 3:

The graph corona of C_n and $k_{1,3}$ is obtained from a cycle C_n by introducing 3 new pendent edges at each vertex of cycle [5]

Definition 4:

Butterfly graph is a graph, when cycles are sharing a common vertex. From the common vertex some pendent edges are attached.

The first results about graph coloring deals almost exclusively with planar graph in the form of the coloring of maps. While trying to color a map of the countries of England Francis Guthrie Postulated the four color conjecture, noting that four colors were sufficient to color the map so that no regions sharing a common border received the same color [3].

Main Results:

Theorem 1:-

We prove Corona of $C_n(n \geq 3)$and $k_{1,3}$ admits a proper coloring, its chromatic number is 2 or 3. The Corona graph of C_n and $k_{1,3}$ is obtained from a cycle C_n by introducing '3' new pendent edges at each vertex of cycle [5].

proof:-

The Proper coloring of C_n will be classified into the following two cases.

Case i)

In the cycle (Number of vertices is an odd) cycle vertices has to be colored as 1, 2 Periodically.

Rest of the single vertex has to be colored as 3.

The $k_{1,3}$ branch vertices which lies on the cycle C_n colored vertex as 1, has to be colored as 2.

Case ii)

In the cycle Cn (Number of vertices is an even) Cycle vertices has to be colored as 1, 2 Periodically.

The $K_{1,3}$ branch vertices which lies on the cycle colored as 1.

Example

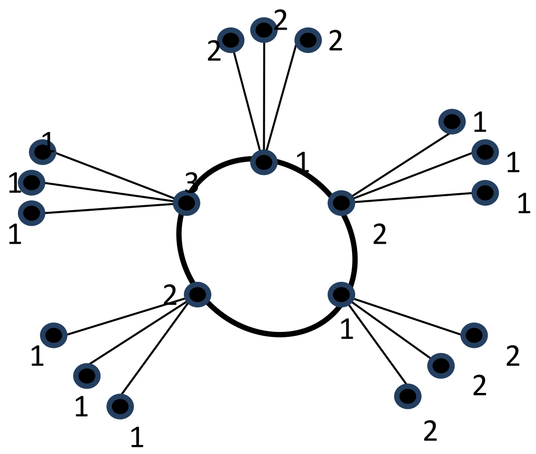

Figure : The graph $C_5\ \Theta_{k1,3}$

Example:-

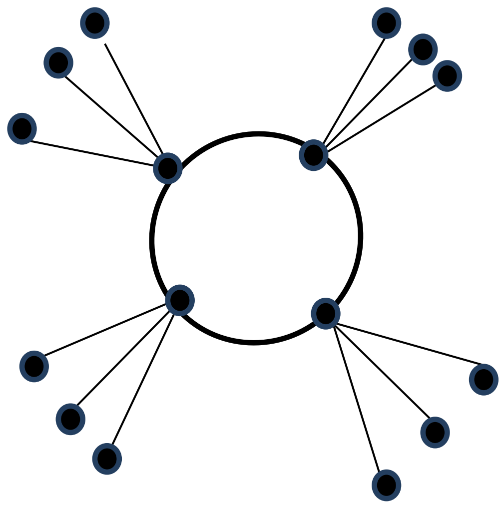

Figure: The graph $C_4 \Theta K_{1,3}$

Theorem 2:-

The graph G is obtained by joining two copies of fan graph Fn by a path of arbitrary length whose proper coloring is 3.

Proof:-

Let G be the graph obtained by joining two copies of fan graph F_n by a path P_K of length K-1 .

Let us denote the successive vertices of first copy of fan graph by $u_1, u_2 ..... u_{n+1}$ and the successive vertices of second copy of fan graph by $w_1, w_2 .... w_{n+1}$. Let $v_1, v_2.....v_k$ be the vertices of path P_K with $v_1= u_1$ and $v_K= w_1$..

Case i)

Number of vertices on the path is an even

i) Path vertices has to be colored as 1, 2, 1, 2 continuously

ii) First copy of the fan graph has to be colored as 2, 3, 2, 3 continuously

iii) Second copy of the fan graph has to be colored as 3, 1, 3 , 1 continuously

Example

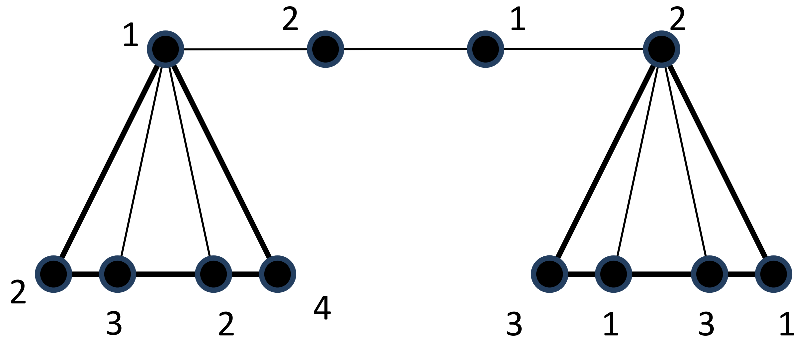

Figure 2 Copies of Fan graph

Case ii)

Number of vertices on the path is an odd.

i) path vertices has to be colored as 1, 2, 1, 2 continuously.

ii) First copy of the Fan graph has to be colored as 2, 3, alternatively.

iii) Second copy of the Fan graph has to be colored as 2,3 alternatively.

Thus its Chromatic number is 3.

Example

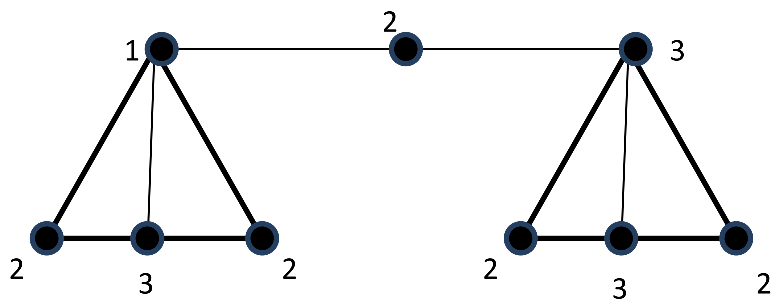

Figure 2 copies of Fan graph

Theorem3;

The Butterfly graph $B_{n,\ m}$ which admits a proper coloring and its chromatic number is 3.

Proof:

Let Bn, m be the butterfly graph with vertices $\{u, vi, wi, xi\}$ Butterfly graph is a graph, when cycle are sharing a common vertex. From the common vertex some pendent edges are attached.

In a butterfly graph, vertices of a cycle is to be colored as,

i) $f(u_1) = 1$

ii) $f\ (vi) =\ 2,3$ periodically, i ≥ 2 [When 'i' is a even]where 'v_i' is left hand side of a butterfly graph.

iii) $f\ (wi) = 2,3$ Periodically, i ≥ 2 [When 'i' is a even] where '$w_{i'}$ is right

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hand side of a butterfly graph.

iv) $f\ (xi) = 3,$ where 'xi's are pendent vertices

Example.

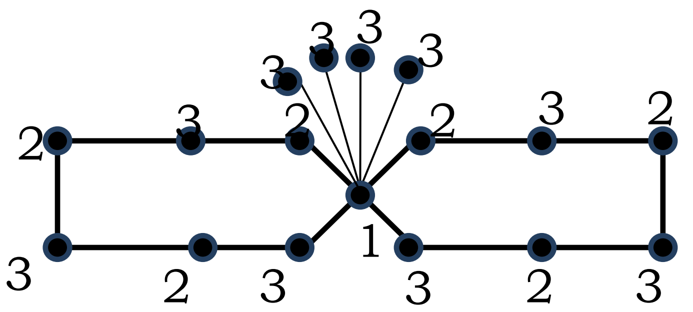

Figure: A butterfly graph

Example

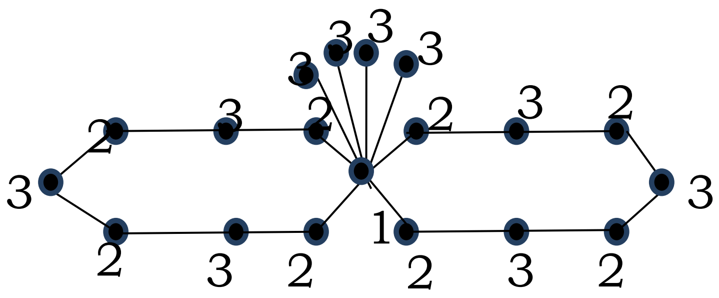

Figure: A butterfly graph

Conclusion:-

In this paper the proper coloring of corona of $C_n \Theta k_{1,3}$ copies of Fan graph and Butterfly graphs were discussed Hence it is of interest to compute the certain classes of graphs, like, subdivision of graphs, Duplication of graphs. etc.,

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