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Research Paper

b-Colouring Of Line Graph and Special Blocks with respect to Path Partition Graph

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Abstract

In Graph theory, the line graph *L(G)* of undirected graph *G* is another graph *L(G)* that represents the adjacencies between the edges of *G*. Other terms used for the line graph are the covering graph, the edge-to-vertex dual, the conjugate, the representative graph, the edge graph, the interchange graph, the adjoint graph and the derived graph.

Mathematics Subject Classification: 05C78; 05C05

Key word

Derived graph, polynomial time, chromatic number

Introduction

All the graphs considered in this paper are finite simple graphs. Terms that are not defined here can be referred from D.B. West, Introduction to Graph Theory, Prentice Hall of India, 2nd Edition, 2001..Suppose $G=(V,E)$ is a graph and $f : E \rightarrow \{1,2,\ldots,|E|\}$ is a bijective

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mapping. For each vertex *u* of *G*, the vertex-sum □ $_f$□□*u*□ at *u* is defined as□□ $_f$□□□*u*□□□□ *f* (*e*) , where *E*(*u*) is the set of edges incident to *u*.

e□*E* (*u*)

If□□ $_f$□□*u*□□□□ $_f$□□*v*□ for any two distinct vertices *u*, *v*□□*V* (*G*) , then *f* is called an anti-magic labeling of *G*. A graph *G* is called anti-magic if *G* has an anti-magic labeling. The problem of anti-magic labeling of graphs was introduced by several authors. They proved that paths, 2-regular graphs and complete graphs are anti-magic and put forth two conjectures concerning anti-magic labeling of graphs.

b-CHROMATIC NUMBER OF LINE GRAPH OF STAR GRAPH $K_{1,n}$

In this section, we study *b*-coloring of line graph of star graph $K_{1,n}$and its chromatic number.

Theorem:

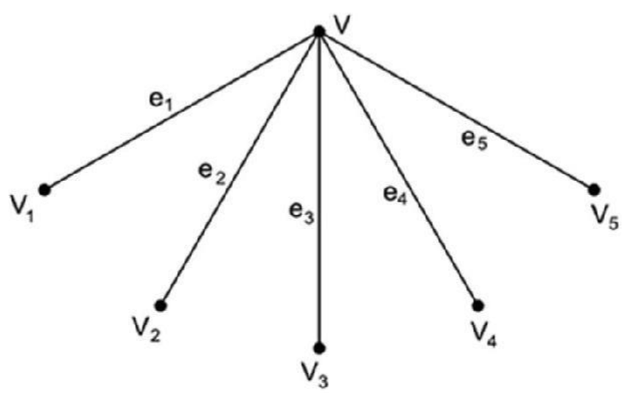
For every n, $\varphi[L(K_{1,n})] = n$.

Proof:

Consider the line graph $K_{1,n}$. The line graph of $K_{1,n}$is a complete graph with *n*-vertices. The *b*-chromatic number of complete graph $K_n$requires *n*-colours for producing a *b*-colouring (fig 1.1).

Therefore $\varphi[L(K_{1,n})] = n$.

Figure 1.1



1.2. *b*-CHROMATIC NUMBER OF LINE GRAPH OF PAN GRAPH

In this section, we discuss *b*-coloring of line graph in pan graph and its chromatic number.

Theorem:

The *b*-chromatic number of every line graph of pan graph is tricolourable.

Proof:

The *n*-pan graph is the graph obtained by joining the cycle graph C_n to $K_1$with a bridge. Consider the line graph of pan graph. By the definition of the line graph, the vertex set of line graph of pan graph corresponds to edge set of the pan graph.

Consider the line graph of pan graph, we see that every line graph of pan graph is a union of cycle C_n with K_3. First we assign the colour to complete graph K_3, by colouring procedure it requires three colours for producing a *b*-chromatic colouring. If we assign any new colour to the cycle C_n, then it does not produce *b*-chromatic colouring because the complete graph K_3do not realizes the new colour. By the colouring procedure the *b*-chromatic number of every Line graph of pan graph is three and is maximum.

A path partition of a graph is a collection of vertex-disjoint paths that cover all vertices of the graph. The path-partition problem is to find the path-partition number p(K) of a graph K , which is the minimum cardinality of a path partition of G. Notice that G has a Hamiltonian path if and only if p(K)=1 Since the Hamiltonian path problem is MP-complete for planar graphs

Suppose every vertex v in the graph G is associated with an integer f(v) ∈ {0, 1, 2, 3}. An f-path partition is a collection P of vertex-disjoint paths such that the following conditions hold.

- Any vertex v with pf(v) ≠2 is in some path in P.
- If pf(v) = 0, then v itself is a path in P.
- If pf(v) = 1, then v is an end vertex of some path in P.

The f-path-partition problem is to determine the f-path-partition number pf (K) which is the minimum cardinality of an f-path partition of G. It is clear that p(K) = pf

(K) when f(v) = 2 for all vertices v in K

2. **Path partition in graphs**

The labeling approach used in this paper starts from the end blocks. Suppose A is an end block whose only cut-vertex is x. Let B be the graph K − (V (A) − {a}). Notice that we can view G as the “composition” of B and A, i.e., G is the union of B and A which meet at a common vertex a. The idea is to get the path-partition number of G from those of A and B. In the lemmas and

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theorems of this paper, we use the following notation. Suppose x is a specified vertex of a graph H in which f is a vertex labeling. For i = 0, 1, 2, 3, we define the function $f_i : V(H) \rightarrow \{0, 1, 2, 3\}$ by $f_i(y) = f(y)$ for all vertices y except fi(x)=i.

Lemma 1

Suppose x is a specified vertex in a graph H. Then the following statements hold.

(1) $pf_3(H) \leq pf_2(H) \leq pf_1(H) \leq pf_0(H)$.

(2) $pf_1(H) \leq pf_0(H) \leq pf_1(H) + 1$.

(3) $pf_2(H) \leq pf_1(H) \leq pf_2(H) + 1$.

(4) $pf_3(H) = \min\{pf_2(H), pf(H - x)\} \leq pf(H - x) = pf_0(H) - 1$.

(5) $pf(H) \geq pf_1(H) - 1$.

Proof.

(1) The inequalities follow from that an f_j-path partition is an f_i-path partition whenever $j < i$.

(2) The second inequality follows from that replacing the path P in an f_2-path partition by two paths P and a results an f_0-path partition of H.

(3) The second inequality follows from that replacing the path P_aQ in an f_1-path partition by two paths P_a and Q results an f_0-path partition of H.

(4) The first equality follows from that one is an f_3-path partition of H if and only if it is either an f_2-path partition of H or an f-path partition of H − a. The second equality follows from that P is an f0-path partition of H if and only if it is the union of {x} and an f-path partition of H – a.

(5) Accordingto(1),(3)and(4),wehavepf(H)≥ $pf_3(H)=\min\{pf_2(H),P_f(H-x)\}\geq\min\{Pf_0(H)-1, P_{f0}(H) - 1\} = P_{f1}(H) - 1$.

Lemma 2

(1) $Pf(G) \le \min\{P_f(A) + P_{f0}(B) - 1, P_{f0}(A) + pf(B) - 1\}$.

(2) $P_{f2}(G) \le P_{f1}(A) + P_{f1}(B) - 1$. Proof. (1) Suppose P is an optimal f-path partition

of A, and Q an f0-path partition of B. Then $x \in Q$ and so $(P \cup Q) - \{x\}$ is an f-path partition of G. This gives $Pf(G) \le Pf(A) + P_{f0}(B) - 1$. Similarly, $P_f(G) \le P_{f0}(A) + P_f(B) - 1$.

(3) The inequality follows from that if P (respectively, Q) is an optimal f1-path partition of A (respectively, B) in which P x ∈ P (respectively, xQ ∈ Q) contains x, then $(P \cup Q \cup \{P\,aQ\}) - \{P\,a, aQ\}$ is an f2-path partition of K.

3. Special blocks:

Notice that the inductive lemma can be applied to solve the path-partition problem on graphs for which the problem can be solved on its blocks. In this paper, we mainly consider the case when the blocks are complete graphs, cycles or complete bipartite graphs. Now, we assume that B is a graph in which each vertex v has a label $f(v) \in \{0, 1, 2, 3\}$. Recall that f (i) is the set of preimages of i,

i.e. $f(i) = \{v \in V(B) : f(v) = i\}$.

According to Lemma 1, we have $Pf(B) = Pf(B - f^{-1}(0)) + |f^{-1}(0)|$.

Therefore, we may assume without loss of generality that $f^{-1}(0) = \emptyset$ throughout this section. We first consider the case when B is a complete graph. The proof of the following lemma is strai ghtforward and hence omitted.

Theorem : For n□□1, the corona product $K_{1,n}$ K_2 are anti-magic.

Proof: Consider a star graph $K_{1,n}$, n□□ 1along with its anti-magic labeling□□ . For the convenience, let v be the central vertex of $K_{1,n}$ and let us name the edges of $K_{1,n}$ as□□e_1 , e_2 , , e_n□□ in such a way that□□□e_i□□□□ i, for 1□□ i□□ n . That is, arrange the edges of $K_{1,n}$ as per the increasing order as defined by the anti-magic labeling□□ . From the definition of anti-magic labeling, the vertex label of a vertex v□□V□□K_n□□ is defined as the sum of the edge labels of edges that are incident with vertex v. Let us arrange the vertices of $K_{1,n}$ as□□v_1 , v_2 , , v_n , v□as per the increasing order of their vertex labels. Since star graphs $K_{1,n}$ are anti-magic, this arrangement of vertices and edges are possible. In the corona product $K_{1,n}$ K_2 , let us name the edges as follows: For the ith vertex v_i of K_n , 1□□ i□□ n, let the edge of K_2 be u_i w_i by

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Vol.11, Iss.9, Dec 2022

considering the vertices of the corresponding copy of K_2 as u_i and w_i . Then, the edges between $K_{1,n}$ and K_2 are denoted as $u_i\ v_i$ and $w_i\ v_i$. Similarly, because of the corona product, for the corresponding copy of the central vertex u, the edges added between the central vertex v and the end vertices of K_2 be u and w. The newly added edges be vu and vw.

Now, let us define the edge labels defined by the function□ . For the original edges of

$K_{1,n}$

□ □e_i□□□□ 3□n□□1□□ i, 1□□ i□□ n .

For the newly added edges in the operation of corona product, we define:

□ □uw□□□ 1

□ □vu□□□ 2

□ □vw□□□ 3

□ □$u_i\ w_i$□□□□ 3□□ i, 1□□ i□□ n

□ □$u_i\ v_i$□□□□ 3□□ n□□ i, 1□□ i□□ n

□ □$w_i\ v_i$□□□□ 4□□ n□□ i, 1□□ i□□ n

It is clear from the definition of□□ , edge labels of edges of $K_{1,n}\ K_2$ are distinct and the edge labels are from the set□□1,2,3, ,4n□□ 3□. It is easy to observe that the vertex

sum of vertices of K_2 defined by□□□ form a monotonically increasing sequence as follows:

□□□□u□,□□□□w□,□□□□u_1□□,□□□□w_1□□,□□□□u_2□□,□□□□w_2□□,

□□□□u_i□□,□□□□w_i□□,

□□□□u_n□□,□□□□w_n□□, followed by

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Vol.11, Iss.9, Dec 2022

the sequence□□□□v_1□□,□□□□v_2□□, ,□□□□v_n□□,□□□□v□.

Therefore, vertex sum defined by□□□ for the vertices of $K_{1,n}$ K_2 are distinct. Thus, for n□□1, the corona product K_2 are anti-magic.

Figure 2-Anti-Magic labeling of $K_{1,6}$ K_2 .

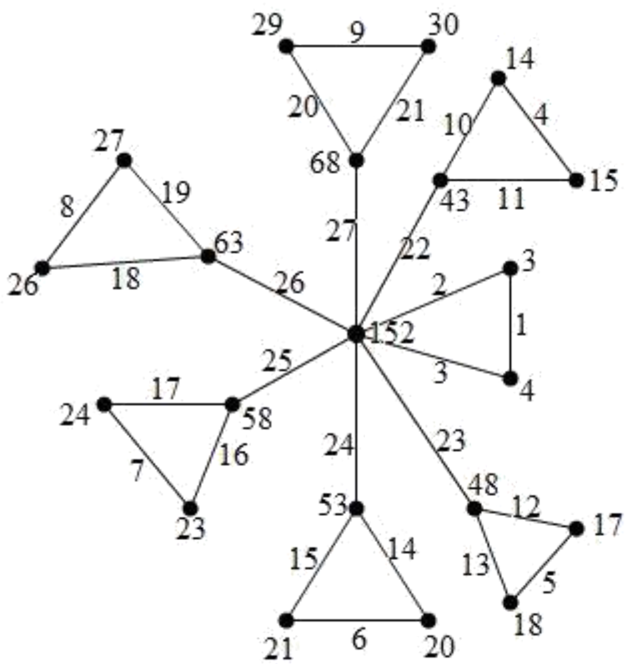

Figure 3: Antimagic labeling of $K_{1,6} \circ K_2$

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