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A STUDY ON INVENTORY MODELS IN MATHEMATICS

*[1] Anandaraj M
M. Phil Scholar, Department of Mathematics,
Bharath University, Chennai. 72
*[2] Dr. K Sivakumar
Professor & Head, Department of Mathematics,
Bharath University, Chennai. 72
anandarajdhusyanth@gmail.com siva111k@gmail.com

Address for Correspondence

*[1] Anandaraj M
M. Phil Scholar, Department of Mathematics,
Bharath University, Chennai. 72
*[2] Dr. K Sivakumar
Professor & Head, Department of Mathematics,
Bharath University, Chennai. 72
anandarajdhusyanth@gmail.com siva111k@gmail.com

Abstract

For a single item, a production inventory model is created. Demand is affected by the amount of inventory on hand and the cost of production. Shortages are permitted and completely booked. The period between making a choice and starting production is known as "preparation time," and it is expected to be crisp or imprecise. The cost of setup is determined by the amount of time it takes to prepare. Using closest interval approximation, the fuzzy preparation time is reduced to a crisp interval preparation time, and the reduced problem is turned to a multi-objective optimization problem using interval arithmetic. For a single-objective crisp model, a mathematical analysis was performed (Model-I). Both crisp (Model-I) and fuzzy (Model-II) models have been numerically shown. Model-I is solved using the generalised reduced gradient approach, whereas Model-II is done using the Global Criteria Method. Some Model-I parameters have been subjected to sensitivity assessments.

Keywords: Fuzzy Preparation Time; Interval Number; Multi-Objective; Global Criteria Method

1. Introduction

After the advancement of EOQ model by Harris [1] in 1915, a great deal of analysts have expanded the above model with various sorts of requests and renewal. A de-followed writing is accessible in the reading material, for example, Hadley and Whitin [2], Tersine [3], Silver and Peterson [4], and so forth In old style stock models, request is nor-mally thought to be consistent. However, presently a-days, with the intrusion of the multi-nationals under the WTO concur ment in the agricultural nations like India, Bangladesh, and so forth, there is a solid contest among the merchants to catch the market for example to appeal the clients through various strategies. Normal practices in such manner is to have appealing beautiful flawless presentation of the things in the show space to make mental strain or to persuade the clients to purchase more. Moreover, de-mand of a thing relies upon unit creation cost, for example it differs conversely with the unit creation cost. This blemish keting strategy is exceptionally helpful for in vogue merchandise/natural products and so on There is some writing on the stock models with stock-subordinate interest. A few creators like Man-dal and Phaujdar [5], Urban [6], Bhunia and Maiti [7], and others concentrated on the previously mentioned sort of stock models.

Presence of Lead-time (the delay between submit ment of request and its real receipt) is a characteristic peculiarity in the field of business. Up until now, the greater part of the scientists have managed either consistent or stochastic lead-time. Rudimentary conversation on lead-time examination are currently a-days accessible in the course readings like Naddor [8], Magson [9], Foote, Kebriaci and Kumin [10] and others. Practically speaking, it is hard to anticipate the lead-time defi-nitely/invaluably and some of the time, the previous records are additionally not accessible to shape a likelihood dispersion for the lead-time. Consequently, the main option accessible to DM is to characterize the lead-time boundary loosely by a fluffy number. By and large, lead time is related with EOQ model for example prompt obtainment or acquisition of the parcel. In any case, in a creation system,the situation is unique. Here the delay between the choice supportive of duction and the genuine beginning of creation mama tters known as planning time for the examination of in-ventory control models. This planning time implies an opportunity to gather the natural substances, to orchestrate talented/un-gifted works, to prepare machine for creation, and so forth, and henceforth impacts the set-up cost of the framework. Interestingly, Mahapatra and Maiti [11,12] planned and tackled creation stock models for a deteri-speaking/fragile thing with loose planning time. regions like air contamination, underlying examination (cf. Rao [13]), transportation (cf. Li and Lai [14]), and so on, till now couple of papers on MODM have been distributed in the field of stock control. Padmanabhan and Vrat [15] formu-lated a stock issue of decaying things with two destinations—minimization of all out normal expense and wastage cost in fresh climate (It is a climate where all info information are thought to be deterministic, pre-cisely characterized and given.) and

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addressed by non-straight objective programming technique. Roy and Maiti [16] defined a stock issue of breaking down things with two objec-tives, specifically, boosting absolute normal benefit and smaller than usual mizing complete waste expense in fluffy climate. Mahapatra, Roy and Maiti [17], Mahapatra, Das, Bhunia and Maiti [18] defined multi-objective multi-thing stock supportive of blems under certain imperatives.

In this paper, a creation stock control framework for a solitary thing is thought of. Here request is depen-imprint on unit creation cost and current stock level. Deficiencies are permitted and multiplied completely. Planning time underway of the new transfer is permitted and fresh/fluffy in nature. The arrangement cost is reliant upon planning time. The fresh issue for limiting affirm age cost is settled by summed up diminished angle method. The issue limiting normal all out cost with fluffy planning time is changed over to a multi-objective minimization issue with the assistance of span number juggling and afterward it is tackled by worldwide measures strategy to get pareto-ideal arrangement. Numerical induction and investigation likewise have been made for both single and multi-objective models. Further, the affectability examination is included and two mathematical representation are given.

2. Interval Arithmetic

Throughout this section lower and upper case letters denote real numbers and closed intervals respectively. The set of all positive real numbers is denoted by R^+, An order pair of brackets defines an interval $A = [a_L, a_R] = \{a: a_L \le a \le a_R, a \in R^+\}$where a_L and a_R are respectively left and right limits of A.

Definition 2.1: Let $* \in \{+, -, ., /\}$ be a binary operation on the set of positive real numbers. If A and B are closed intervals then $A*B = \{a*b: a \in A, b \in B\}$ defines a binary operation on the set of closed intervals. In the case of division, it is assumed that $0 \notin B$. The operations on intervals used in this paper may be explicitly calculated from the above definition as

$$\frac{A}{B} = \frac{[a_L, a_R]}{[b_L, b_R]} = [\frac{a_L}{b_R}, \frac{a_R}{b_L}]$$

where $0 \notin B, 0 \le a_L \le a_R$ and $0 < b_L \le b_R$

$$A + B = [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R]$$

$$kA = \begin{cases} [ka_L, ka_R], & \text{for } k \ge 0 \\ (ka_R, ka_L), & \text{for } k < 0, \quad k \text{ is a real number} \end{cases}$$

order Relations between intervals:

Here, the orders relations which represent the decision- maker's preference between interval costs are defined for minimization problems. Let the uncertain costs for two alternative

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be represented by intervals. A and B respectively. It is assusmed that the cost of each alternative is known only to lie to the corresponding interval. The order relation by the left and right limits of interval is defined in Definition-2.2

Definition 2.2: The order relation $\leq_{LR}$ between $A = [a_L, a_R]$ and $B = [b_L, b_R]$ is defined as

$$A \leq_{LR} B \text{ if } a_L \leq b_L \text{ and } a_R \leq b_R$$
$$A <_{LR} B \text{ if } A \leq_{LR} B \text{ and } a_R \neq b_R$$

The order relation $\leq_{LR}$ represents the DM's performance for the altemative with the lower minimum cost, that is, if $A \leq_{LR} B$, then A is preferred to B.

2.2. Formulation of the Multi-Objective Problem

A general non-linear objective function with some interval valued parameters is as follows:

$$\text{Minimize } Z(x) = \sum_{i=1}^{k} C_i \prod_{j=1}^{n} x_j^{a_j}$$
$$\text{subject to } \sum_{i=1}^{k} A_1 x_j \leq B_j$$
$$x_j > 0, (j = 1,2, \cdots, n.), x = (x_1, x_2, \cdots, x_n)$$

where $C_i = [c_L, c_{pA}], A_1 = [a_{Li}, a_{Ri}]$ and $B_j = [b_{Lj}, b_{Ry}]$.

Now, we exhibit the formulation of the original problem (1) as a multi-objective non-linear problem. Since the objective function $Z(x)$ and the constraints contain some parameters represented by intervals, it is natural that the solution set of (1) should be defined by preference relations between intervals.

Now from Equation (1) the right and left limits, $z_{白(x),zL}(x)$ and centre $z_C(x)$ of the interval objective function $Z(x)$ respectively may be elicited as

$$z_R(x) = \sum_{i=1}^{k} c_L \prod_{j=1}^{n} x_j^{2y}, z_L(x) = \sum_{i=1}^{k} c_{Li} \prod_{j=1}^{n} x_{jy}^{a_y}$$
$$z_C(x) = \frac{1}{2}[z_R(x) + z_l(x)]$$

Thus the problem (1) is transformed into

Minimize $\{z_R, z_C\}$

subject to $\sum_{i=1}^{k} a_{Li} x_j \leq b_{\text{py}}$

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Vol.11, Iss.9, Dec 2022

$$\sum_{i=1}^{k} a_{Re}x_j \leq b_{lj}$$
$$x_j > 0, (j = 1,2, \cdots, n), x = (x_1, x_2, \cdots, x_n)$$

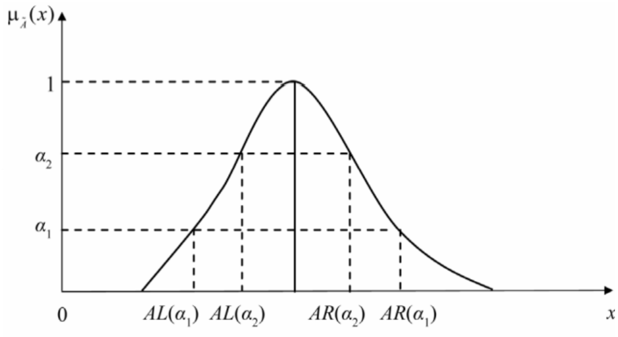

Figure 1. Fuzzy number $A^{\tilde{}}$ with α-cuts.

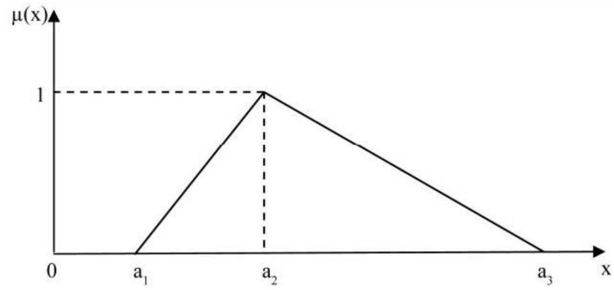

Figure 2. Triangular fuzzy number.

$$\mathrm{d}(\hat{A}, \tilde{B}) = \sqrt{\int_0^1 (A_2(\alpha) - B_L(\alpha))^2 \, \mathrm{d}\alpha + \int_0^1 (A_R(\alpha) - B_p(\alpha))^2 \mathrm{d}\alpha}$$

Given A is a fuzzy number. We have to find a closed interval $C_d(A^{\tilde{}})$ which is nearest to λ with respect to metric d. We can do it since each interval is also a fuzzy number with constant α-cut for all $\alpha \in [0,1]$. Hence $(C_d(A^{\tilde{}}))_\alpha = [C_L, C_R]$. Now we have to minimize

$$\mathrm{d}(A^{\tilde{}}, C_d(A^{\tilde{}})) = \sqrt{\int_0^1 (A_L(\alpha) - C_L(\alpha))^2 \, \mathrm{d}\alpha + \int_0^1 (A_R(\alpha) - C_R(\alpha))^2 \, \mathrm{d}\alpha}$$

with respect to C_l and $C_{原}$. In order to minimize $\mathrm{d}(\lambda, C_d(A^{\tilde{}}))$, it is sufficient to minimize the function $D(C_L, C_R) = (\mathrm{d}^2(A^{\tilde{}}, C_d(A^{\tilde{}})))$. The first partial derivatives are

$$\frac{\partial D(C_L, C_R)}{\partial C_L} = -2 \int_0^1 A_L(\alpha) \mathrm{d}\alpha + 2C_L$$

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und $\frac{\partial D(C_L,C_R)}{\partial C_R} = -2\int_0^1 A_R(\alpha)\mathrm{d}\alpha + 2C_s$

olving $\frac{\partial D(C_L,C_R)}{\partial C_L} = 0$ and $\frac{\partial D(C_L,C_g)}{\partial C_R} = 0$

we get $C_L^* = \int_0^1 A_L(\alpha)\mathrm{d}\alpha$ and $C_R^* = \int_0^1 A_R(\alpha)\mathrm{d}\alpha$.

Again since.

4. Mathematical Formulation

Production inventory system involves only one item. A cycle starts with shortages at time $t = 0$ and at time t_1 maximum shortages level is Q_s and at that time pro- duction process starts to backlog the shortage quantities fully and after time t_2 the shortages reached to zero, inventory accumulates up to time t_3 of amount Q_m. At that time production process stopped, accumulated inventory declines due to demand and reaches to zero at time t_0. The above production inventory system is shown in **Figure 2**. The differential equations governing the stock status for this model are given by.

5.Conclusion:

The present paper proposes a solution procedure for production inventory model with production cost and on hand inventory dependent demand rate and preparation time. Here, shortages are allowed and backlogged fully. Like lead time, time when the decision is taken for the preparation of next production run *i.e.* preparation time has been considered for a production inventory model. In real life, setup cost decreases with the increase of prepa- ration time. This consideration is taken into account in Model-I. In Model-II, preparation time is taken as im- precise via a fuzzy number. The fuzzy number is described by linear/non-linear type membership function.

Fuzzy number describing preparation time is then approximated to an interval number. Following this, the problem is converted into multi-objective inventory problem where the objective functions are represented by centre and right limit of interval function which are to be minimized. To obtain the solution of the multi-objective inventory problem, Global Criteria Method has been used. The proposed demand has a broad area of applicability. Demand of a commodity decreases with the in- crease in production cost but increases with the increase of stock of the displayed commodity and vice versa. Here, though the formulation of the model and the solution procedure are quite general, the model is a simple production model with demand dependent production rate. The unit production cost which is assumed here to be constant, in reality, varies with the preparation time and produced quantity. Moreover, time dependent production rate, partially lost sales, inflation, etc., can be incorporated to the model to make it more realistic. Here, demand is stock- dependent. The present analysis can be repeated for the dynamic demand also. Though the problem has been presented in crisp and fuzzy environment, it can also be formulated and solved in fuzzy-stochastic environment representing production cost and inventory costs through probability distribution.

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