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Research Paper

A view on Labeling of Star Related Graphs

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ABSTRACT

Star graph is a special type of graph in which n-1 vertices have degree 1 and a single vertex has degree n-1. This looks like n-q vertex is connected to a single central vertex. A star graph with total n-1 vertex is termed as S_n. A star graph is a complete bipartite graph if a single vertex belongs to one set and all the remaining vertices belong to the other set.

KEYWORDS

Star graph, vertices, trees, vertex, edge, pair sum, etc.

Introduction

A Star tree of order n is a tree with as many leaves a possible or in other words a star tree is a tree that consists of a single internal vertex n-1 and n-1 leaves. However, an internal vertex is a vertex of degree at least 2. Nodes that have no child are called leaf nodes. Here we prove that some trees which are obtained from stars are pair sum.

Theorem

The trees G_j, $(1\leq j\leq 5)$ with vertex set and edge set given below are pair sum.

(i) $V(G_1)= V(K_{1,m})\cup\{y_j:1\leq j\leq 6\}$ and

$E(G_1)= E(K_{1,m})\cup\{xy_1, y_1y_2,y_2y_3,y_3y_4,y_4y_5,y_5y_6\}$.Then G_1 is a pair sum graph.

(ii) $V(G_2)= V(K_{1,m})\cup\{y_j:1\leq j\leq 7\}$

and$E(G_2)=E(K_{1,m})\cup\{xy_6,y_6y_7,y_1y_2,y_2y_3,y_3y_4,y_4y_5,y_5x\}$.

Then G_2 is a pair sum graph.

(iii) $V(G_3) = V(K_{1,m}) \cup \{y_j: 1 \leq j \leq 7\}$ and

$E(G_3) = E(K_{1,m}) \cup \{xy_5,y_5y_6,y_6y_7,y_1y_2,y_2y_3,y_3y_4,y_4x\}$

Then G_3 is a pair sum.

(iv) $V(G_4) = V(K_{1,m}) \cup \{y_j z_j: 1 \leq j \leq 4\}$ and

$E(G_4) = E(K_{1,m}) \cup \{xz_1,z_1z_2,xz_3,z_3z_4,y_1y_2,y_2y_3,y_3y_4,y_4x\}$.

Then G_4 is a pair sum graph.

(v) $V(G_5) = V(K_{1,m})\cup\{y_jz_j:1\leq j\leq 3\}$

and $E(G_5)=E(K_{1,m})\cup\{xy_1,xy_2,xy_3,y_1z_1,y_2z_2,y_3z_3\}$.Then G_5 is a pair sum graph.

Proof (i)

$V(K_{1,m})=\{x,x_j:1\leq j\leq m\}$ and

$E(K_{1,m})=\{xx_j 1\leq j\leq m\}$.

$g: V(G_1)\rightarrow\{\pm1,\pm2,\ldots,\pm(m+7)\}$

by $g(x)=-1$, $g(y_1)=-7$, $g(y_2)=-5$,

$g(y_3)=1$, $g(y_4)=3$, $g(y_5)=5$,

$g(y_6)=7$ $g(x_j)=-2j-4$, $1\leq j\leq\lceil \frac{m}{2}\rceil$and

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$g(x_{\lfloor (m+1)/2+j \rfloor}) = 2i+6$, $1 \le j \le \lceil \frac{m}{2} \rceil$

Then G_1 is a pair sum tree.

Proof (ii)

Define a map

$g: V(G_2) \rightarrow \{\pm1, \pm2, \ldots, \pm(m+8)$

by $g(y_1) = -3$, $g(y_2) = -6$, $g(y_3) = -1$,

$g(y_4) = -4$, $g(y_5) = 1$, $g(y_6) = 3$,

$g(y_7) = 4$, $g(x) = 2$, $g(x_1) = 7$,

$g(x_{1+j}) = 4+2i$, $1 \le j \le \lfloor \frac{m}{2} \rfloor$ and

$g(x_{\lceil (m/2)+j \rceil + j}) = -2j-8$, $1 \le j \le \lceil \frac{m-2}{2} \rceil$

Proof (iii)

Define a map

$g: V(G_3) \rightarrow \{\pm1, \pm2, \ldots, \pm(m+8)\}$

$g(y_1) = -3$, $g(y_2) = -6$, $g(y_3) = -1$,

$g(y_4) = -4$, $g(y_5) = 2$, $g(y_6) = 3$,

$g(y_7) = 4$, $g(x) = 1$,

$g(x_1) = 7+j$, $1 \le j \le \lceil \frac{m}{2} \rceil$

$g(x_{\lceil (m/2) \rceil + j}) = -j-10$, $1 \le j \le \lfloor \frac{m}{2} \rfloor$

Then G_3 is a pair sum graph.

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Proof (iv)

Define a map

$g:V(G_4)\rightarrow\{\pm1,\pm2,\ldots,\pm(m+9)\}$

by $g(x)=-1, g(y_1)=3, g(y_2)=2,$

$g(y_3)=1, g(y_4)=-4, g(z_1)=-5,$

$g(z_2)=-6, g(z_3)=7,$

$g(z_4)=4$. For the other vertices we define,

$g(x_j)=-5-2j,\ 1\le j\le\lceil \frac{m}{2}\rceil$

$g(x_{\lfloor (m+1)/2\rfloor+j})=7+2j,\ 1\le j\le\lfloor \frac{m}{2}\rfloor.$

Obviously g is a pair sum labeling.

Proof (v)

Define

$g: V(G_5)\rightarrow\{\pm1,\pm2,\ldots,\pm(m+7)\}$

by $g(x)=-1, g(y_1)=2, g(y_2)=3,$

$g(y_3)=4, g(z_1)=-3, g(z_2)=-5,$

$g(z_3)=-7,$

$g(x_j)=2j+4,\ 1\le j\le\lceil \frac{m}{2}\rceil$

$g(x_{\lfloor (m+1)\rfloor+j})=-2-2j,\ 1\le j\le\lfloor \frac{m}{2}\rfloor.$

Obviously g is a pair sum labeling.

Theorem

Let G be the tree with $V(G) = V(B_{n,m}) \cup \{z_j : 1 \le j \le 6\}$

and $E(G) = E(B_{n,m}) \cup \{yz_1, z_1 z_2, z_2 z_3, yz_4, z_4 z_5, z_5 z_6\}$.

Then G is a pair sum graph.

Proof

Define $V(B_{n,m}) = \{x, y, y_i, v_i : 1 \le i \le n, 1 \le i \le m\}$ and

$E(B_{n,m}) = \{xy, xx_i, yy_j : 1 \le i \le n, 1 \le j \le m\}$.

$g : V(G) \to \{\pm 1, \pm 2, \ldots, \pm(n+m+8)\}$ and

$g(z_1) = 4$, $g(z_2) = 5$, $g(z_3) = 6$,

$g(z_4) = -2$, $g(z_5) = -7$, $g(z_6) = -4$,

$g(x) = 2$, $g(y) = -1$.

Case 1): n=m $g(x_1) = -3$,

$g(x_{1+j}) = 9 + j$, $1 \le j \le m - 1$,

$g(y_j) = -10 - j$, $1 \le j \le m$

case 2): n>m

Assign the label to x_j, $y_j (1 \le j \le m)$ as in case 1.

Define $g(x_{m+j}) = 8 + m + j$, $1 \le j \le \lceil \frac{n-m}{2} \rceil$ and

$g(x_{\lceil (n+m)/2 \rceil + j}) = -13 - m - j$, $1 \le j \le \lfloor \frac{n-m}{2} \rfloor$.

case 3): n<m

Assign the label to x_j, $y_j (1 \le j \le n)$ as in case 1.

Define $g(x_{n+j})=11+n+j,\quad 1\le j\le \lceil \frac{m-n}{2} \rceil$

$g(x_{\lceil (n+m)/2 \rceil + j}) = -10-n-j,\ 1\le j \le \lfloor \frac{m-n}{2} \rfloor$.

Then G is a pair sum graph.

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Theorem

If G is the tree with $V(G)=V(B_{n,m}) \cup \{z_j: 1\le j\le 3\}$ and

$$E(G)=E(B_{n,m})\cup\{xz_1, z_1z_2, z_2z_3, z_3y\}/\{xy\}.$$

Then G is a pair sum tree.

Proof

Define a function $g: V(G)\to\{\pm1,\pm2,\ldots,\pm(n+m+5)\}$ βψ

$g(x)=-1, g(y)=3, g(z_1)=-4,$

$g(z_2)=1, g(z_3)=2.$

Case 1): $n=m$.

$g(x_j)=-5-j,\ 1\le j\le m$ and

$g(y_i)=3+j,\ 1\le j\le m.$

case 2): n>m

Assign the label to $x_j, y_j (1\le j\le m)$ as in case 1.

Define $g(x_{m+j})=-5-m-j,\ 1\le j\le \lceil \frac{n-m}{2} \rceil$

and $g(x_{\lfloor (n+m)/2 \rfloor + j})=7+m+j,\ 1\le j\le \lfloor \frac{n-m}{2} \rfloor$.

Then G is a pair sum graph

Theorem

Let G be the tree with $V(G)= V(B_{n,m})\cup\{z_j : 1\le j\le 4\}$ and

$E(G)= E(B_{n,m})\cup\{xz_1, z_1z_2, z_2y, yz_3, z_3z_4\}/\{xy\}$. Then G is a pair sum graph.

Proof

Define $g: V(G)\to\{\pm1, \pm2,\dots, \pm(n+m+6)\}$

by $g(x)=-1$, $g(y)=2$, $g(z_1)=-4$,

$g(z_2)=1$, $g(z_3)=3$, $g(z_4)=4$.

Case 1): $n=m$.

$g(x_1)=-6$,

$g(x_{j+1})=-6-j$, $1\le j\le m-1$

and $g(y_j)=5+j$, $1\le j\le m$.

case 2): n>m

Assign the label to x_j, $y_j (1\le j\le m)$ as in case 1.

Define $g(x_{m+j})=-5-m-j$, $1\le j\le \lceil \frac{n-m}{2} \rceil$ and

$g(x_{\lceil (n+m)/2\rceil+j})= 8+m+j$, $1\le j\le \lfloor \frac{n-m}{2} \rfloor$.

case 3): n<m

Assign the label to x_j, $y_j (1\le j\le m)$ as in case 1.

Define $g(y_{m+j})=-8-n-j$, $1\le j\le \lceil \frac{m-n}{2} \rceil$ *and*

$g(y_{\lceil (n+m)/2\rceil+j})= 5+n+j$, $1\le j\le \lfloor \frac{m-n}{2} \rfloor$.

Then G is a pair sum graph.

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