e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

Research Paper

AN EQUITABLE COLORING OF DIFFERENT CLASSES OF GRAPHS

KARUPPIAH. S
Research Scholar
M.Phil Mathematics
Bharath Institute Of Higher Education And Research

Dr. N. RAMYA
Head of the Department, Department Of Mathematics
Bharath Institute Of Higher Education And Research

Address for Correspondence:
KARUPPIAH. S
Research Scholar
M.Phil Mathematics
Bharath Institute Of Higher Education And Research
Mail Id : karuppiahbhuvana@gmail.com

INTRODUCTION

The branch of mathematics concerned with networks of points connected by lines is known as graph theory. It has been discovered numerous times independently from a variety of leisure math challenges. Indeed, the subject is first mentioned in the work of Euler (1707-1782), the Father of Graph Theory and Topology, who answered a renowned unsolvable problem of his day called the Konigsberg Bridge problem in 1736. Euler proved the first theorem in graph theory by solving this riddle.For the next 100 years, nothing was done in the field after Euler's work. G.R Kirchhoff (1824-1887) created the tree theory in 1847 to solve the system of simultaneous linear equations that determine the current in each branch and circuit of an electrical network. Cayley (1821-1895) developed the significant class of graphs known as trees ten years later by using differential calculus to study the change of variables. Later, he worked on enumerating the isomers of the saturated hydrocarbons C_nH_{2n+2}, which have a fixed number of carbon atoms.Jordan (1869) was the first to recognise trees as a purely mathematical study. In addition,

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

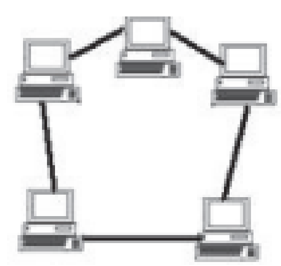
the game devised by Sir William Hamilton in 1859 gave rise to the key Graph Theory concepts of Hamiltonian pathways and Hamiltonian cycles.

How Graph Theory is Connected to Other Subjects?

Graph theory was discovered by the realm of theoretical research for its own purposes. The points represent molecules in Uhlenbeck's statistical mechanics investigation, and two neighbouring points suggest nearest neighbour interaction of some type (for example the magnetic attraction or repulsion). Another application of graph theory in physics is as a visual aid. The figure proposed by Feynmann has points representing physical particles and lines representing particle pathways after collisions.

Planarity has profound chemical ramifications in 19th century chemistry, where lettered vertices signify individual atoms and connecting lines denote chemical bonds (with degree corresponding to valence). The 19th century Englishman James Sylvester, one of numerous mathematicians interested in counting certain sorts of diagrams representing molecules, is credited with the first usage of the term graph theory in this context.

For example, the many computers in a building and peripheral devices such as printers and plotters can be connected via local area networks based on any of the following strategies. 1. The network of stars 2. The ring system 3. A combination of the first two..



Star network Ring network

Figure 1.1: Some Local Area Networks

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

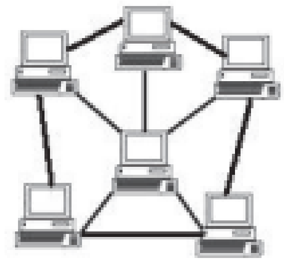


Figure 1.2: Hybrid of star and ring networks

In a parallel algorithm, a single instruction stream directs the input and output of sub problems to the appropriate processors and regulates the execution of the algorithm by delivering sub problems to different processors. When using parallel processing, one CPU may require output from another processor. As a result, these processors must be linked together. As a result, we can utilise the proper graph type to represent the interconnection networks. Below is a list of the most regularly used types..

1.1.4 A Two-way Link Between Each Pair of Processors

This is represented by K_n, the complete graph on n vertices.

1.1.5 Linear Array

Each processor P_i, i = 2, 3, .. . ,n−1 is connected to its neighbors P_{i-1} and P_{i+1} through a two-way link. P_1 is connected to P_2 and P_n is connected to P_{n-1}.

Figure 1.3: A linear array for six processors.

1.1.6 The Mesh Network (Two Dimensional Array)

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

In such a network the number of processors is a perfect square, say $n = m^2$. The n processors are labeled as P(i, j) for $(0 \leq i \leq m - 1)$, $(0 \leq j \leq m - 1)$. Two way links connect processors P(i, j) as shown in figure for n = 16.

GRAPH COLORINGS

THE ORIGIN OF CHROMATIC GRAPH THEORY

The origins of chromatic graph theory can be traced back to 1852, when Augustus De Morgan wrote a letter to his friend William Rowan Hamilton, informing him that one of his students had noticed that only four colours were required to colour a map of England, ensuring that neighbouring countries could be assigned different colours. Hamilton was uninterested in the four-color conjecture, which claimed that any map could be coloured with four colours in this fashion. This nugget of mathematical knowledge was practically forgotten until C.S. Peirce delivered a proof in a Harvard seminar in the 1860s. The conjecture gained traction in 1878, when Arthur Cayley inquired whether it had been settled at a meeting of the London Mathematical Society. Cayley quickly followed up with a short message outlining the problem and pointing out the problems. A few months later, a putative proof of the four-color conjecture emerged in the newly created American Journal of Mathematics. This was A. B. Kempe's famous erroneous proof, which lasted over ten years before the error was revealed.

In 1880, the first paper with edge colorings was published. P.G.Tait, Professor of Natural Philosophy at the University of Edinburgh, outlined some additional proofs of the four-color conjecture in these papers, and deduced that the edges of every cubic map can be coloured with only three colours in such a way that the three edges meeting at each vertex are assigned different colours. He also asserted the reverse result without proof, claiming (incorrectly) that the edges of every cubic map may be coloured with three colours by induction, resulting in a straightforward proof of the four-color conjecture.

P.J. Haewood presented an important study in 1890 rejecting Kempe's demonstration and demonstrating that any map's countries may be coloured properly with five colours. He followed up eight years later with the first in a series of papers in which he reconstructed Tait's concepts in

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

terms of congruences. The idea behind this method is to assign one of the numbers +1 or 1 to each vertex of a cubic map so that the sum of the numbers around each region of the map equals 0. (mod3). The four-color conjecture is true, according to Haewood, if and only if every set of congruences obtained in this method has a non-trivial solution. Following Julius Petersen's work on graph factorization, D. K. onig established that if G is any bipartite graph or multigraph and its greatest valency is, then the edges of G may be coloured with exactly colours in such a way that all of the edges meeting at any vertex are coloured differently.

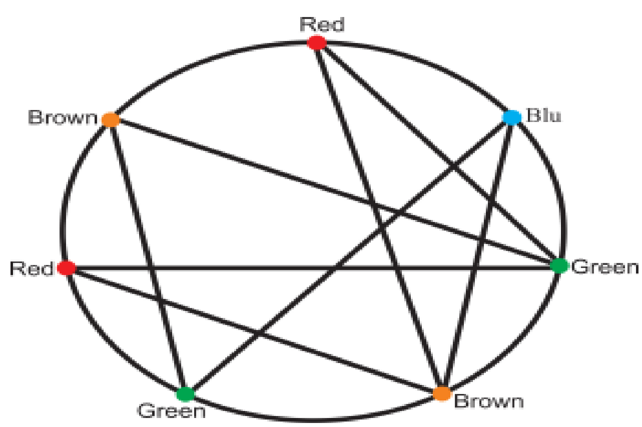
Scheduling of Final Exams

A graph model can be used to address the scheduling problem, with vertices representing courses and edges connecting them if there is a common student in the courses they represent. Each hue represents a distinct time window for a final exam. The colouring of the accompanying graph corresponds to the exam timetable.

Consider the following scenario: there are seven finals to be planned. Assume the classes are numbered 1 through 7. Assume that the students in the following pairs of classes are the same..

1 and 2, 1 and 3, 1 and 4, 1 and 7.

2 and 3, 2 and 4, 2 and 5, 2 and 7.



.

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

Figure 2.1: Graph model showing scheduling.

Since the chromatic number of the above graph is 4, four time slots are needed. One can arrange them as follows:

Time Period	Courses
I	1-6
II	2
III	3-5
IV	4-7

Let G = (V (G), E(G)) be a graph with V as the vertex set and E as the edge set. The vertices and edges of a graph G are coloured with k colours so that no two neighbouring or incident elements have the same colour. The least k such that G has a k-total colouring is the total chromatic number ″(G) of G. If the number of vertices and edges coloured with each hue differs by no more than one, the overall colouring is equitable [18, 26, 28, 36, 51, 53]. Equitable total chromatic number, represented by ″=, is the smallest k for which G has such a colouring (G).

The difficulty of eq uitable division of a system with binary conflicting relations into conflict-free sub-systems can occur in some discrete industrial systems. Equitable graph colouring can be used to model such scenarios.

For example, in the garbage collection problem [52], the vertices of the graph represent garbage collection routes, and pairs of vertices are connected by an edge if the corresponding routes should not be run on the same day. As a result, the problem of assigning one of the six days of the work week to each route reduces to the problem of graph colouring. In practise, having a roughly equal number of routes run on each of the six days could be preferable. As a result, six colours must be used to colour the graph in an impartial manner.

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

Scheduling the timetabling is another application of equitable edge-coloring. For example, if we want to build university timetables with the same amount of lectures for every class and every instructor every day, we must first construct a bipartite graph G(X, Y), where X is the set of vertices relating to teachers and Y is the set of vertices pertaining to classes. If and only if the class y has lectures with teacher x, two vertices x X and y Y are connected by an edge. The difficulty of determining a "equitable" schedule is reduced to a fair edge-coloring of G. (X, Y).

Certain apparent results may be seen: I =(Kn) = n, (ii) =(Kn,n) = 2, (iii) =(Pn) = 2 and hence (iv) = (L(Pn)) = 2.

.The Equitable Δ-Coloring Conjecture

In contrast to common appropriate graph colouring, equitable colouring lacks monotonicity; that is, a graph can be equitable k-colorable without being equitable (k + 1)-colorable. As a result, the ECC hides the true nature of the equitable coloration. It appears that the maximum degree is important in this case. Chen, Lih, and [9] proposed the following after reviewing relevant evidence.

Let G be a connected graph, and the Equitable-Coloring Conjecture (ECC) be true. G is equitably (G)-colorable if it is not a complete graph, an odd cycle, or a complete bipartite graph K2m+1,2m+1.

The ECC's conclusion is equivalently written as = (G) (G). The fact that the ECC implies the ECC is likewise obvious. If the ECC is valid for regular graphs, it is also true for non-regular graphs. The ECC was established by Chen, Lih, and Wu [7] for graphs with a maximum degree of at least one-half of the order. The basic tools for the answer were provided by the following two lemmas. G is used to represent G's complement graph. The lowest degree and edge-independence number of G are denoted by (G) and ′(G), respectively.

2.6 MISCELLANEOUS RESULTS

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

Let $K_{n_1,n_2,\ldots,n_t}$ denote the complete t−partite graph whose parts have sizes $n_1, n_2, \cdots, n_t$. Chen and Wu [12] proved the following two theorems.

Theorem 2.6.1. $\chi_{=}\left(K_{n_1,n_2,\cdots,n_t}\right) = \sum_{i=1}^{t} \lceil n_i/h \rceil$ *where* $= \max\left\{k : n_i \geq \lceil n_i/k \rceil (k-1)\right.$ *for all* $\left. i\right\}$.

Theorem 2.6.2. $\chi^*_{=}\left(K_{n_1,n_2,\cdots,n_t}\right) = \sum_{i=1}^{t} \lceil n_i/h \rceil$ *where* $= \min\{k :$ *there is* i *such that* $n_i < \lceil n_i/(k+1) \rceil k$ *or there are* n_i *and* n_j, $i \neq j$, *such that* k *divides neither* n_i *nor* $n_j\}$.

Take two graphs G_1 and G_2. The cartesian product of the vertex sets $V(G_1) \times V(G_2)$ to be the vertex set of a new graph. There are several ways to define the edge set of a product graph. Let us introduce two products [40], the square product G_1 G_2 and the cross product $G_1 \times G_2$. They are also known as the cartesian and the direct products, respectively. The edge sets are defined as follows.

$E(G_1 \square G_2) = \{(u, x)(v, y) : (u = v \text{ and } xy \in E(G_2)) \text{ or } (x = y \text{ and } uv \in E(G_1))\}$, $E(G_1 \times G_2) = \{(u, x)(v, y) : uv \in E(G_1) \text{ and } xy \in E(G_2)\}$.

The following results concerning products are included in a note by Chen, Lih and Yan[10]

EQUITABLE COLORING OF CORONA PRODUCT OF GRAPHS K_n, C_n AND P_n

Let us look at the equitable colouring of various corona products G H of two graphs G and H in this chapter. Even if G is 4-regular and H is K2, determining the colorability of G H is NP-complete. The equitable chromatic number =(G H), where G is an equitably 4-colorable graph and H is a complete graph, a cycle, or a path, is then given specific values or upper bounds. This confirms the Equitable Coloring Conjecture for corona products of these graph classes.

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

Meyer [48] popularised the concept of fair colorability. However, Hajnal and Szemer'edi [25] already shown that a graph G of degree is equitably k-colorable if k(G) + 1. For such colouring, Mydlarz and Szemeredi'edi [49] discovered a polynomial-time method. Kierstead and Kos tochka [39] have provided a brief demonstration of this bound as well as another polynomial-time method. Meyer [48] proposed the following hypothesis in 1973:

Equitable Coloring Conjecture (ECC) is Conjecture 3.2.1. Other than a full graph or an odd cycle, for every linked graph G, =(G) (G).

For any graphs with six or fewer vertices, this hypothesis has been confirmed. The Equitable Coloring Conjecture is true for all bipartite graphs, according to Lih and Wu [44]. Wang and Zhang [55] studied r-partite graphs, which are a wider class of graphs. Meyer's conjecture is valid for entire graphs from this class, they demonstrated. For outerplanar graphs [60] and planar graphs with a maximum degree of at least 13 [58], the hypothesis was also verified.

A simple reduction from graph colouring to equitable colouring may be shown by adding a sufficient number of isolated vertices to a graph, demonstrating that testing if a graph has an equitable colouring with a given number of colours is NP-complete (greater than two). For trees (previously known thanks to Chen and Lih [8]) and outerplanar graphs, Bodlaender and Fomin [2] demonstrated that equitable colouring may be solved to optimality in polynomial time. For equitable colouring of split graphs, a polynomial time method is also known [6].

The graph G H created from one copy of G and |V (G)| copies of H, where the ith vertex of G is next to every vertex in the ith copy of H, is called the corona of two graphs G and H. Frucht and Harary [17] were the first to offer such graph products in 1970. The corona Pn K1 is a comb graph, for example. Figure 3.1 shows another corona graph, namely L(K4)K2, where L(G) is a line graph of graph G. (b) The rest of this chapter is laid out as follows: Let's begin the following part with a theory on the difficulty of equitable corona colouring. Even for the corona of line graphs of cubic graphs (i.e. 3-regular) and K2, the issue turns out to be NP-hard. Corona products of graphs G with =(G) 4 and cycles are studied in Section 3.4. The corona products of the graphs G and routes are investigated in Section 3.5. Let us confirm the ECC conjecture by

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

establishing a new class of graphs that can be coloured optimally in polynomial time. Finally, in Section 3.6, we summarise our findings, which also applies to bipartite graphs.

NP-COMPLETENESS PROOF

FINAL REMARKS

The chapter concludes with an NP-completeness proof for equitable colouring of corona graphs. For certain specific instances of such products, a poloynomial time solution for equitable colouring is also established. Of course, the difficulty of equitable 3- or 4-coloring of graph G, which is usually NP-hard, affects the difficulty of equitable colouring of GH. However, the following graphs: broken spoke wheels, reels, and certain graph products, as well as the associated coronas, allow equitable 3-coloring in polynomial time [19, 20, 46]. In addition, the Equitable Coloring Conjecture has been verified for these graphs. The following is a summary of our findings. Table 3.1.

G \ H		even cycles C_{2k}		odd cycles	paths P_k	
		$k = 2$	$k \geq 3$		$2 \leq k \leq 5$	$k \geq 6$
equitably 3-colorable graph G on $n \geq 2$ vertices	$3 \mid n$	3	3	4	3	3
	$3 \nmid n$		4			4
equitably 4-colorable graph G on $n \geq 2$ vertices	$3 \nmid n$	3	4	4	≤ 4	4

Table 3.1: Values of the equitable chromatic number of coronas $G \circ H$.

Finaly, note that some of our methods can be extended to equitable coloring of other classes of graphs. For example the algorithm given in the proof of Theorem 3.4.1 is also good for coloring bipartite graphs.

Theorem 3.6.1. Let G be an equitably 3-colorable graph on n ≥ 2 vertices and let H be a bipartite graph. If 3|n then $\chi_=(G \circ H) \leq 3$. Moreover, if H has at least one edge then $\chi_=(G \circ H) = 3$.

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

Proof. Let us color the graph G equitably with 3 colors. After that let us the order vertices of G: $v_1, v_2, \ldots, v_n$ in such a way that vertex v_i is colored with color i mod 3 and, as previously, use color 3 instead of color 0.

Let us assume that our bipartite graphs $H = H(V_1, V_2)$. Coloring the ith, $i = 1, \ldots, n$, copy of bipartite graph H using $|V_1(H)|$ times color ((i mod 3) + 1) mod 3 and $|V_2(H)|$ times color ((i mod 3) + 2) mod 3.

In the coloring each of three colors is used exactly $1 + |V_1(H)| + |V_2(H)|$ times. This means that our coloring is equitable.

EQUITABLE COLORING OF CENTRAL GRAPH AND TOTAL GRAPH OF SOME FAMILIES OF GRAPHS

In this chapter, interesting results regarding the equitable chromatic num ber $\chi_=$ for the central graph of star graph $C(K_{1,n})$, the central graph of complete bigraph $C(K_{n,n})$, the central graph of complete graph $C(K_n)$, the central graph of cycle $C(C_n)$, the central graph of path $C(P_n)$, the total graph of complete bigraph $T(K_{m,n})$, the total graph of path $T(P_n)$ and the total graph of cycle $T(C_n)$.

4.1 EQUITABLE COLORING OF CENTRAL GRAPH OF $K_{1,n}$, $K_{n,n}$, K_n, C_n AND P_n

Theorem 4.1.1. The equitable chromatic number of central graph of star graph, $\chi_=(C(K_{1,n})) = n$.

Proof. Let $V(K_{1,n}) = \{v, v_1, v_2, \ldots, v_n\}$ where $vv_i = e_i$ $(1 \leq i \leq n)$ by the definition of Central graph $C(K_1, n)$ has the vertex set $V(K_{1,n}) \cup \{u_i: 1 \leq i \leq n\}$ where u_i is the vertex of

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

subdivision of the edge e_i. Also the vertex subset $\{v_1, v_2, . . . v_n\}$ of $K_{1,n}$ induces a clique on n vertices.

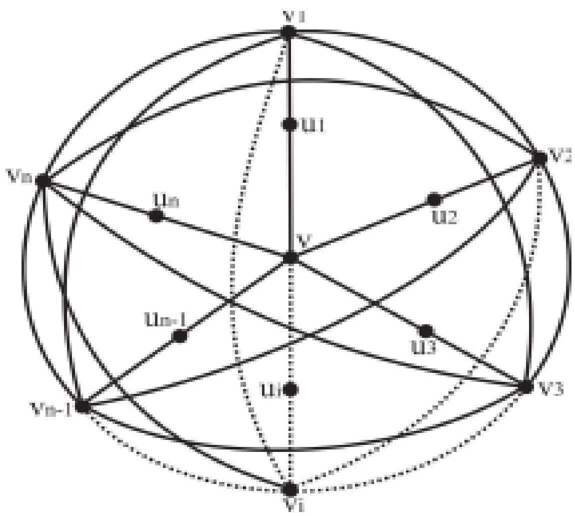


Figure 4.1: Central graph of Star graph C ($K_{1,n}$).

Now partition the vertex set V (C ($K_{1,n}$)) as follows.

$V_1 = \{v, v_1\}$

$V_2 = \{u_1, v_2, u_n\}$

$V_i = \{v_i, u_{i-1} : 3 \leq i \leq n\}$.

Clearly V_1, V_2 and V_i $(3 \leq i \leq n)$ are independent sets of C ($K_{1,n}$). Also, $|V_1| = |V_i| = 2$ $(3 \leq i \leq n)$ and $|V_2| = 3$. It holds the inequality $||V_i| - |V_j|| \leq 1$ for i 6= j. To prove: $\chi_=$ (C ($K_{1,n}$)) $\leq$ n. Assume that $\chi_=$ (C ($K_{1,n}$)) < n, Since C ($K_{1,n}$) contains a clique of order n, χ (C ($K_{1,n}$)) $\geq$ n and so $\chi_=$ (C ($K_{1,n}$)) $\geq$ n. This contradiction shows that $\chi_=$ (C ($K_{1,n}$)) $\geq$ n. Hence $\chi_=$ (C ($K_{1,n}$)) = n.

EQUITABLE COLORING OF TOTAL GRAPH OF $K_{m,n}$, P_n AND C_n

Theorem 4.2.1. If m $\leq$ n, the equitable chromatic number of total graph of com plete bigraphs,

$$\chi_{=}(T(K_{m,n})) = \begin{cases} n+1 \;\; if\, m < n \\ n+2 \;\; if\, m = n \end{cases}$$

Proof. Let (X, Y) be the bipartition of $K_{m,n}$ where X = {v_i: 1 ≤ i ≤ m}, and Y = $v'_{j:\,1\leq j\leq n}$. Let u_{ij} (1 ≤ i ≤ m, 1 ≤ j ≤ n) are the edges of $v_i v'_j$. By the defini tion of total graph T ($K_{m,n}$) has the vertex set {$v_{i:\,1\leq i\leq m}$}∪ $v'_{j:\,1\leq j\leq n}$ ∪ {u_{ij} : 1 ≤ i ≤ m, 1 ≤ j ≤ n} and the vertices {u_{ij} : 1 ≤ i ≤ m, 1 ≤ j ≤ n} induce n disjoint cliques of order n in T ($K_{m,n}$). Also v_i (1 ≤ i ≤ m) is adjacent to v'_j(1 ≤ j ≤ n).

Case 1: m = n,

Now partition the vertex set V (T($K_{m,n}$)) as follows,

V_1 = {u_{11}, u_{2n}, $u_{3(n-1)}$, $u_{4(n-2)}$, ..., $u_{(n-1)3}$, u_{n2}}

V_2 = {u_{12}, u_{21}, u_{3n}, $u_{4(n-1)}$, ..., $u_{(n-1)4}$, u_{n3}}

. . . .

V_n = {u_{1n}, $u_{2(n-1)}$, $u_{3(n-2)}$, $u_{4(n-3)}$, ..., $u_{(n-1)3}$, u_{n1}}

V_{n+1} = {v_1, v_2, . . . , v_n}

V_{n+2} = {v'_1, v'_2, ..., v'_n}.

Clearly V_1, V_2, . . . , V_{n+2} are independent sets and |V_i| = n (1 ≤ i ≤ n + 2) sat isfying the condition ||V_i| − |V_j|| = 0, for any i 6= j, $\chi_{=}$ (T ($K_{m,n}$)) ≤ n + 2. Since there exist a clique of order n + 1 in T ($K_{m,n}$). χ (T($K_{m,n}$) ≥ n + 1. Also each v_i of T ($K_{m,n}$) receives one color different from the color class assigned to the clique induced by {u_{ij} : 1 ≤ i ≤ m, 1 ≤ j ≤ n}. By the definition of total graph each $v_i$is adjacent with v'_j(1 ≤ j ≤ n). Therefore {v_1, v_2, . . . , v_m} and {v'_1, v'_2, . . . , v'_n} are independent sets and hence χ (T ($K_{m,n}$)) ≥ n + 2. That is $\chi_{=}$ (T ($K_{m,n}$)) ≥ χ (T ($K_{m,n}$)) ≥ n + 2, therefore $\chi_{=}$ (T ($K_{m,n}$)) ≥ n + 2. Hence $\chi_{=}$ (T ($K_{m,n}$)) = n + 2.

Case 2: If m < n

Now we partition the vertex set $V(T(K_{m,n}))$ as follows,

$V_1 = \{u_{11}, u_{22}, u_{33}, u_{44}, \ldots, u_{mm}\} \cup \{v'_n\}$

$V_2 = u_{12}, u_{23}, u_{34}, \ldots, u_{m(m-1)} \cup \{u_{m1}\} \cup \{v'_1\}$

$V_3 = u_{13}, u_{24}, u_{35}, \ldots, u_{m(m-2)} \cup u_{(m-1)3}, u_{m2} \cup \{v'_2\}$

. . . .

. . . .

$V_{n-1} = u_{1(n-1)}, u_{2n}\} \cup \{u_{31}, u_{32}, \ldots, u_{m(m-2)} \cup v'_{n-2}$

$V_n = \{u_{1n}\} \cup u_{21}, u_{32}, \ldots, u_{m(m-1)} \cup v'_{n-1}$

$V_{n+1} = \{v_1, v_2, v_3, \ldots, v_m\}$.

Clearly $V_1, V_2, \ldots, V_{n+1}$ are independent sets of $T(K_{m,n})$. Also $|V_1| = |V_2| = \ldots = |V_n| = m + 1$ and $|V_{n+1}| = m$ satisfy the condition $||V_i| - |V_j|| \leq 1$, for any i 6= j, $\chi_= (T(K_{m,n})) \leq n + 1$. Since there exist a clique of order n + 1 in $T(K_{m,n})$. $\chi(T(K_{m,n})) \geq n + 1$, that is $\chi_= (T(K_{m,n})) \geq \chi(T(K_{m,n})) \geq n + 1$,

therefore $\chi_= (T(K_{m,n})) \geq n + 1$. Hence $\chi_= (T(K_{m,n})) = n + 1$.

Theorem 4.2.2. The equitable chromatic number of total graph of path, $\chi_= (T(P_n)) = 3$.

Proof. Since $T(P_n)$ contains at least one cycle of length 3, we conclude that $\chi_= (T(P_n)) \geq 3$. Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and let $V(T(P_n)) = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n - 1\}$. Where u_i is the vertex of $T(P_n)$ corresponding to edge $v_i v_{i+1}$ of P_n.

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.9, Dec 2022

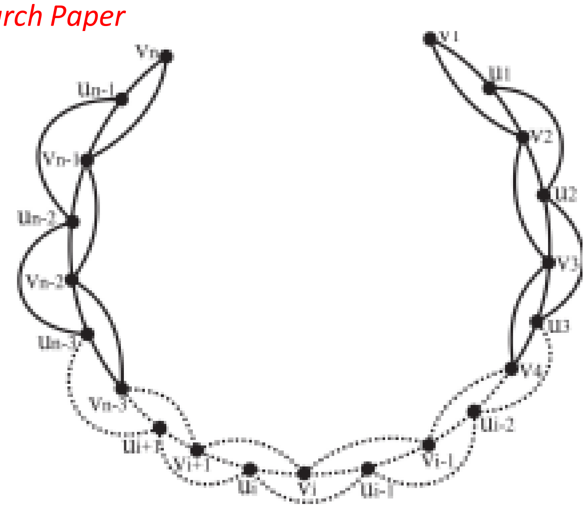


Figure 4.4: Total graph of Path T (P_n).

Case 1: n ≡ 0 mod 3

Consider the following independent sets of T (P_n).

$V_1 = \{v_i: i \equiv 1 \bmod 3, 1 \leq i \leq n-2\} \cup \{u_j: j \equiv 2 \bmod 3, 2 \leq j \leq n-1\}$

$V_2 = \{v_i: i \equiv 2 \bmod 3, 2 \leq i \leq n-1\} \cup \{u_j: j \equiv 0 \bmod 3, 3 \leq j \leq n-3\}$

$V_3 = \{v_i: i \equiv 3 \bmod 3, 3 \leq i \leq n\} \cup \{u_j: j \equiv 1 \bmod 3, 1 \leq j \leq n-2\}$

Clearly $V(T(P_n)) = \bigcup_{i=1}^{3} V_i'$'s are mutually disjoint such that $|V_1| = \frac{2n}{3}, |V_2| = \frac{2n}{3}, |V_3| = \frac{2n}{3}$. Therefore there exist a partition (v_1, v_2, v_3) of $V(T(P_n))$ such that $||V_i| - |V_j||$ for $i \neq j$ for $n \equiv 0 \bmod 3$.

Case 2: n ≡ 1 mod 3

Consider the following independent sets,

$V_1 = \{v_i: i \equiv 1 \bmod 3, 1 \leq i \leq n\} \cup \{u_j: j \equiv 2 \bmod 3, 2 \leq j \leq n-2\}$

$V_2 = \{v_i: i \equiv 2 \bmod 3, 2 \leq i \leq n-2\} \cup \{u_j: j \equiv 0 \bmod 3, 3 \leq j \leq n-1\}$

$V_3 = \{v_i: i \equiv 0 \bmod 3, 3 \le i \le n-1\} \cup \{u_j: j \equiv 1 \bmod 3, 1 \le j \le n-3\}$

The above said sets V_1, V_2, V_3 are disjoint independent sets of $V(T(P_n))$, such that $|V_1| = \frac{2n+1}{3}, |V_2| = \frac{2n-3}{3}, |V_3| = \frac{2n-2}{3}$. Clearly $||V_i| - |V_j|| \le 1$ for $i \ne j$.

Case 3: $n \equiv 2 \bmod 3$,

Consider the following independent sets,

$V_1 = \{v_i: i \equiv 1 \bmod 3, 1 \le i \le n-1\} \cup \{u_j: j \equiv 2 \bmod 3, 2 \le j \le n-3\}$

$V_2 = \{v_i: i \equiv 2 \bmod 3, 2 \le i \le n\} \cup \{u_j: j \equiv 0 \bmod 3, 3 \le j \le n-2\}$

$V_3 = \{v_i: i \equiv 0 \bmod 3, 3 \le i \le n-2\} \cup \{u_j: j \equiv 1 \bmod 3, 1 \le j \le n-1\}$

Clearly the above said partition satisfies the condition, $||V_i| - |V_j|| \le 1$ for

$i \ne j$. Therefore $\chi_{=}(T((P_n)) = 3$.

Conclusion:

Graph theory was discovered by the realm of theoretical research for its own purposes. The points represent molecules in Uhlenbeck's statistical mechanics investigation, and two neighbouring points suggest nearest neighbour interaction of some type (for example the magnetic attraction or repulsion). Another application of graph theory in physics is as a visual aid. The figure proposed by Feynmann has points representing physical particles and lines representing particle pathways after collisions.Planarity has profound chemical ramifications in 19th century chemistry, where lettered vertices signify individual atoms and connecting lines denote chemical bonds (with degree corresponding to valence). The 19th century Englishman James Sylvester, one of numerous mathematicians interested in counting certain sorts of diagrams representing molecules, is credited with the first usage of the term graph theory in this context. When pottery samples are supplied at an archaeological dig, we want to know what styles were utilised when, so we create interval graphs. Assume that each type was used for a specific period of time, and that two styles found in the same grave were employed at the same time. Then, if possible, its interval representation is the possible time lines

REFERENCES

[1] Bermond.J.C., Harutyunyan.H.A., Arthur L., Liestman., Stephane Perennes, A Note on the Dimensionality of Modified Kn¨odel Graphs, International J. of Foundations of Computer Science, Vol.8(2), pp.109–116, 1997.

[2] Bodleander.H.L., Fomin.F.V., Equitable colorings of bounded treewidth graphs, Theor. Comput. Sci., Vol.349(1), pp.22–30, 2005.

[3] Bollob´as and Guy.R.K., Equitable and proportional coloring of trees, J. Combin. Theory Ser. B, Vol.34, pp.177-186, 1983.

[4] Bondy.J.A and Murty.U.S.R, Graph theory with Applications. London: MacMillan 1976.

[5] Brooks.R.L., On coloring the nodes of a network, Proc. Cambridge Philos. Soc., Vol.37, 194-197, 1941.

[6] Chen.B.L., Ko.M.T and Lih.K.W., Equitable and m-bounded coloring of split graphs, in: Combinatorics and Computer Science (Brest, 1995) LCNS 1120, pp.1-5, Springer 1996.

[7] Chen.B.L., Lih.K.W., A note on the m-bounded chromatic number of a tree, J. Combinatorics, Vol.14, pp.311-312, 1993.

[8] Chen.B.L and Lih.K.W., Equitable coloring of trees, J. Combin. Theory Ser. B Vol.61, pp.83-87, 1994.

[9] Chen.B.L., Lih.K.W and Wu.P.L., Equitable coloring and the maximum degree, Europ. J. Combinatorics Vol.15, pp.443-447, 1994.

[10] Chen.B.L., Lih.K.W and Yan.J.H., Equitable coloring of graph products, manuscript, 1998.

[11] Chen.B.L., Lih.K.W and Yan.J.H., A note on equitable coloring of interval graphs, manuscript, 1998.

[12] Chen.B.L and Wu.C.H., The equitable coloring of complete partite graphs, manuscript, 1994.

[13] Cohen.J., Pierre Fraigniaud, Cyril Gavoille, Recognizing Knodel graphs ¨ , Discrete Mathematics, Vol.250, pp.41-62, 2002.

[14] Danuta Michalak., On middle and total graphs with coarseness number equal 1, Springer Verlag Graph Theory, Lagow proceedings, Berlin Heidelberg, New York, Tokyo, 139–150, 1981.

[15] Fraigniaud.P and Peters.J.G., Minimum Linear Gossip Graphs and Maximal Linear (Delta,k)-Gossip Graphs, Networks, Vol.38, pp.150-162, 2001.

[16] Frank Harary, Graph Theory, Narosa Publishing home - 1969.

[17] Frucht.R., Harary.F., On the corona of two graphs, Aequationes Math. Vol.4, pp.322–325, 1970.

[18] Fu.H.L., Some results on equalized total coloring, Congressus Numerantium, Vol.102, pp.111-119, 1994.

[19] Furmaczyk.H., Equitable coloring of graphs, in: Graph Colorings, (M. Kubale, ed.) American Mathematical Society Providence, Rhode Island 2004.

[20] Furmaczyk.H., Equitable coloring of graph products, Opuscula Mathematica Vol.26(1), pp.31–44, 2006.