Proper coloring of Octopus graph, Cubic graph Barbell graph and Drum graph

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Abstract:

The notion of proper coloring of graphs are discussed below. We give the exact value of chromatic number for, Octopus graph, Barbell graph, Cubic graph and Drum graph are discussed.

Introduction

Graph theory is an old subject with many modern applications. Its basic ideas were introduced in the eighteenth century by the great Swiss mathematician Leonhard Euler. In the last three decades graph theory has established itself a worthwhile mathematical discipline and there are many applications of graph theory to a wide variety of subjects such as operation research, physics etc. [5]

Preliminaries

Definition 1

A simple graph G is called a regular graph if each vertex of G has an equal degree.

A regular graph G is called a cubic graph if all the vertices of G are of degree 3.[2]

Definition 2

The Drums graph D_n, $n \geq 3$ can be constructed by two cycle graphs $2C_n$, $n \geq 3$ joining two path graphs $2P_n$, $n \geq 2$ with sharing a common vertex. i.e., $D_n = 2C_n + 2P_n$.[3]

Definition 3

An Octopus graph O_n, $n \geq 2$ can be constructed by a Fan graph F_n, $n \geq 2$ joining a star graph $K_{1,n}$ with sharing a common vertex, where n is any positive integer. i.e., $O_n = F_n + K_{1,n}$.[4]

Main results

Theorem 1:

An octopus graph O_n ($n \geq 2$) holds proper coloring and whose chromatic number is 3 [4].

Proof:

Let G be an octopus graph O_n.

Let $\{u_1\ u_2 \ldots u_{2n+1}\}$ be the vertices of O_n.

u_1 be the central vertex of O_n.

u_{1+i}, (where i = 1, 2, … n) are uppermost vertices.

u_{n+j} (where j = 1, 2, … n) are the pendant vertices.

Let $f : V(G) \rightarrow \{1, 2, 3\}$ is defined as follows,

(i) $f(u_1) = 3$
(ii) $f(u_{1+i}) = 1$, when i = 1, 3, 5, … n–1
(iii) $f(u_{1+i}) = 1$, when i = 1, 3, 5, … n–1
(iv) $f(u_{n+j}) = 1$ for all j

where ‘n’ represents number of upper vertices in the Fan graph.

Illustration:

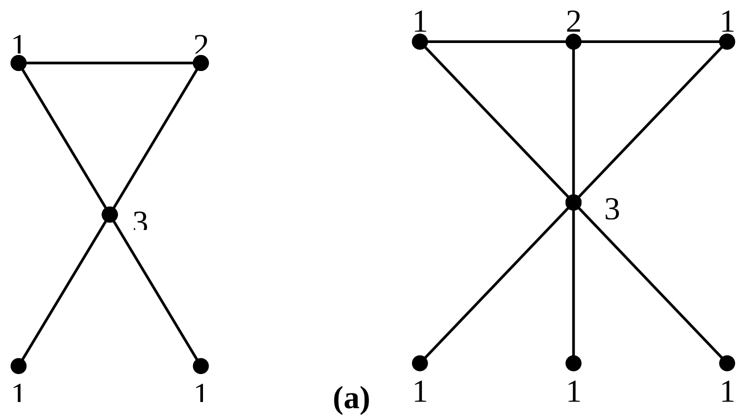

Figure 1: (a) Octobus graph O_2 (b) Octobus graph O_3

Theorem 2:

A path union of two copies of cubic graph establishes proper coloring, and chromatic number is 2 [2].

Proof:

Let G = (V, E) be a cubic graph.

Let V(G) = $\{u_1, u_2, u_3, u_4, u_5, u_6, v_1\ v_2\ v_3\ v_4\ v_5\ v_6\}$

E(G) = $\{u_i\, u_{i+1} / 1 \le i \le 5, u_1\, u_6\} \cup \{v_i\, v_{i+1} / 1 \le i \le 5, v_1\, v_6\} \cup \{u_i\, v_i / 1 \le i \le 6\}$

Gp be the path union of two copies of cubic graph of G and G′ respectively.

Define the coloring f : V(G) $\rightarrow$ {1, 2, 3} is classified in to three cases.

In the cubic graph $u_1, u_2 \ldots u_n$ be the inner vertices and $v_1\ v_2 \ldots v_n$ be the outer vertices of G.

Similarly $u_1'\ u_2' \ldots u_n'$ be the inner vertices of G′ and $v_1'\ v_2' \ldots v_n'$ be the outer vertices of G′.

Case (i): When the vertices (inner or outer) vertices of any cycle, n = 3, 6, 9 … 3k (where k = 1, 2 …) coloring has to be given for G is

(i) $f(u_i) = 1$, when i = 1, 4, 7 … k–2

(ii) $f(u_i) = 2$, when i = 2, 5, 8 … k–1

(iii) $f(u_i) = 3$, when i = 3, 6, 9 … k

(iv) $f(v_i) = 1$, when i = 3, 6, 9 … k

(v) $f(v_i) = 2$, when i = 1, 4, 7 … k–2

(vi) $f(v_i) = 3$, when i = 2, 5, 8 … k–1

Similarly coloring of G′ has to be

(i) $f(u_i') = 1$, when i = 3, 6, 9 … k

(ii) $f(u_i') = 2$, when i = 1, 4, 7 … k–2

(iii) $f(u_i') = 3$, when i = 2, 5, 8 … k–1

(iv) $f(v_i') = 1$, when i = 2, 5, 8 … k–1

(v) $f(v_i') = 2$, when i = 3, 6, 9 … k

(vi) $f(v_i') = 3$, when i = 4, 7, 10 … k–2

Illustration 1:

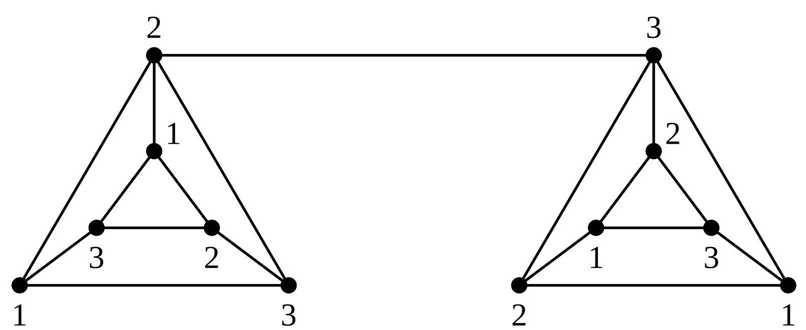

Figure 2: Path union of two copies of cubic graph with 12 vertices

Illustration 2:

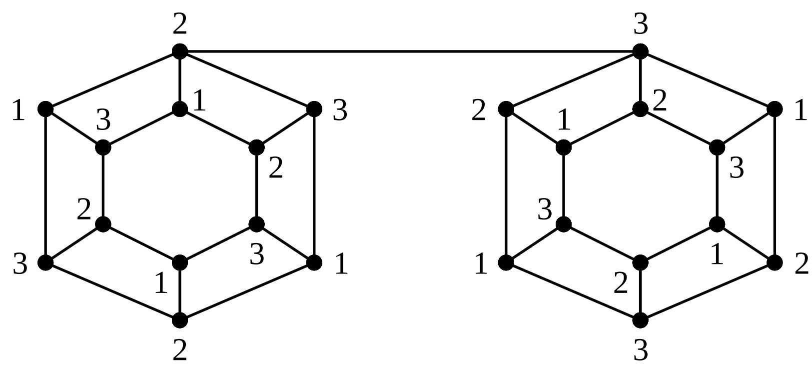

Figure 3: Path union of two copies of cubic graph with 24 vertices

Case (ii): When the vertices (inner or outer) of any cycle, n = 1, 4, 7 … 3k−2 (where k = 1, 2 …) coloring has to be given for G is

(i) $f(u_i) = 1$, when i = 1, 4, 7 … 3j–2 (j = 1, 2, …)

(ii) $f(u_i) = 2$, when i = 2, 5, 8 … 3j–1 (j = 1, 2, …)

(iii) $f(u_i) = 3$, when i = 3, 6, 9 … 3j (j = 1, 2, …)

(iv) $f(u_i) = 2$, when i = n–1

(v) $f(v_i) = 1$, when i = 3, 6, 9 … 3j (j = 1, 2, …)

(vi) $f(v_i) = 2$, when i = 1, 4, 7 … 3j–2 (j = 1, 2, …)

(vii) $f(v_i) = 3$, when i = 2, 5, 8 … 3j–1 (j = 1, 2, …)

(viii) $f(v_i) = 3$, when i = n

Similarly coloring of G′ has to be

(i) $f(u_i') = 3$, when i = 3, 6, … 3j (j = 1, 2, …)

(ii) $f(u_i') = 1$, when i = 1, 4, 7 … 3j–2 (j = 1, 2, …)

(iii) $f(u_i') = 2$, when i = 2, 5, 8 … 3j–1 (j = 1, 2, …)

(iv) $f(u_i') = 2$, when i = n

(v) $f(v_i') = 2$, when i = 1, 4, 7 … 3j–2

(vi) $f(v_i') = 3$, when i = 2, 5, 8 … 3j–1

(vii) $f(v_i') = 1$, when i = 3, 6, 9 … 3j

(viii) $f(v_i') = 3$, when i = n

Illustration 1:

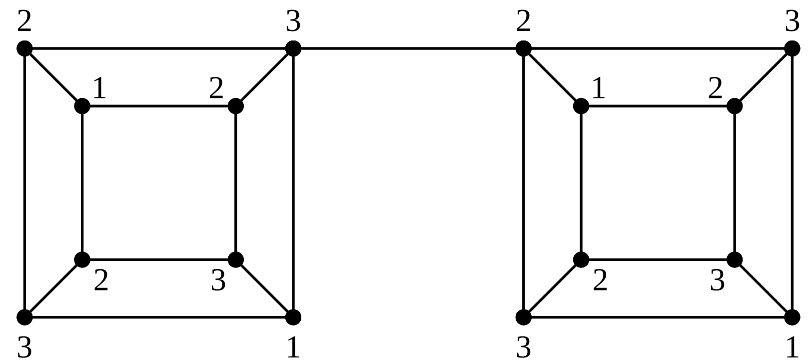

Figure 4: Path union of two copies of cubic graph with 16 vertices

Illustration 2:

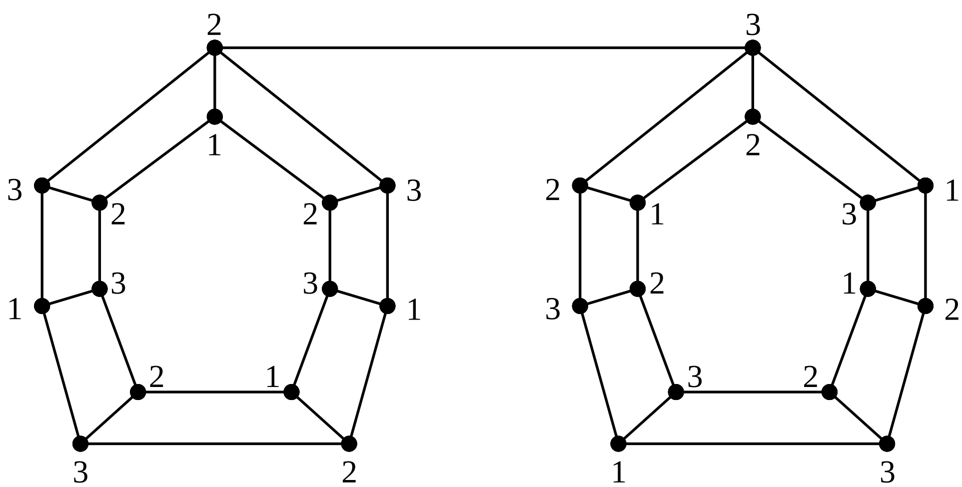

Figure 5: Path union of two copies of cubic graph with 28 vertices

Case (iii): When the vertices (inner or outer) of any cycle, n = 2, 5, 8 … 3k–1 (where k = 1, 2 …) coloring has to be given for G is

(i) $f(u_i) = 1$, for i = 1, 4, 7 … 3j–2 (j = 1, 2, …)

(ii) $f(u_i) = 2$, for i = 2, 5, 8 … 3j–1 (j = 1, 2, …)

(iii) $f(u_i) = 3$, for i = 3, 6, 9 … 3j (j = 1, 2, …)

(iv) $f(u_i) = 1$, for i = n–1

(v) $f(u_i) = 2$, for i = n

(vi) $f(v_i) = 1$, for i = 3, 6, 9 … 3j (j = 1, 2, …)

(vii) $f(v_i) = 2$, for i = 1, 4, 7 … 3j–2 (j = 1, 2, …)

(viii) $f(v_i) = 3$, for i = 2, 5, 8 … 3j–1 (j = 1, 2, …)

(ix) $f(v_i) = 2$, for i = n–1

(x) $f(v_i) = 3$, for i = n

Coloring of G′ has to be given as,

(i) $f(u_i') = 2$, for i = 1, 4, 7 … 3j–2 (j = 1, 2, …)

(ii) $f(u_i') = 3$, for i = 2, 5, 8 … 3j–1 (j = 1, 2, …)

(iii) $f(u_i') = 1$, for i = 3, 6, 9 … 3j (j = 1, 2, …)

(iv) $f(u_i') = 2$, for i = n–1

(v) $f(u_i') = 3$, for i = n

(vi) $f(v_i') = 1$, for i = 2, 5, 8 … 3j–1 (j = 1, 2, …)

(vii) $f(v_i') = 2$, for i = 3, 6, 9 … 3j (j = 1, 2, …)

(viii) $f(v_i') = 3$, for i = 1, 4, 7 … 3j–2 (j = 1, 2, …)

(ix) $f(v_i') = 1$, for i = n

(x) $f(v_i') = 3$, for i = n–1

Illustration:

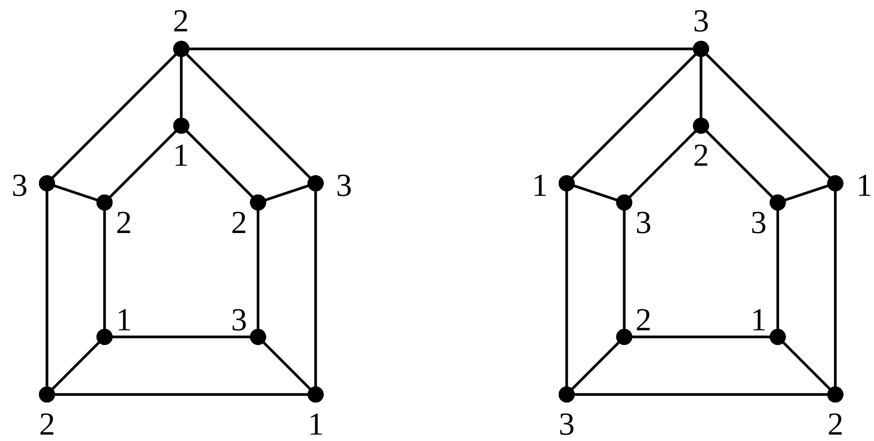

Figure 6: Path union of two copies of cubic graph with 20 vertices

Theorem 3:

A barbell graph B(m, n) which holds proper coloring and whose chromatic number is n, where n represents number of vertices of a complete graph [1]

Proof:

Let B(m, n) be the Barbell graph, and let $c_{11}, c_{12}, c_{13} \ldots c_{1n}$ be the set of vertices of the first cycle.

$c_{21}, c_{22}, c_{23} \ldots c_{2n}$ be the set of vertices of the second cycle.

$c_{31}, c_{32}, c_{33} \ldots c_{3n}$ be the set of vertices of the third cycle.

In general $c_{m1}, c_{m2} \ldots c_{mn}$ be the set of vertices of the m^{th} cycle.

Coloring has to be given, $f : V(G) \to \{1, 2 \ldots n\}$ is defined as follows

(i) $f(c_{m1}) = 1$
(ii) $f(c_{m2}) = 2$
(iii) $f(c_{m3}) = 3$
(iv) $f(c_{m4}) = 4$

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(v) $f(c_{m5}) = 5$

…

(vi) $f(c_{mn}) = n$

Illustration 1:

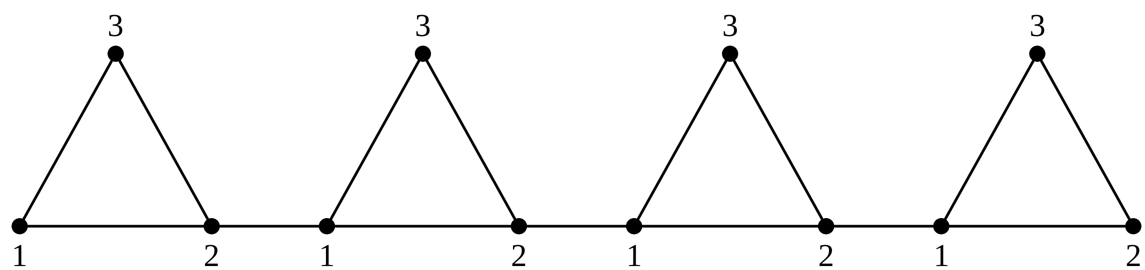

Figure 7: Barbell B(4, 3) - graph

Illustration 2:

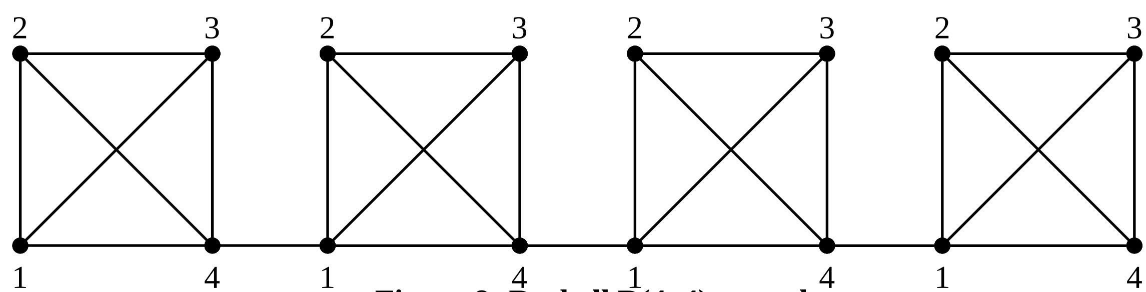

Figure 8: Barbell B(4, 4) - graph

Illustration 3:

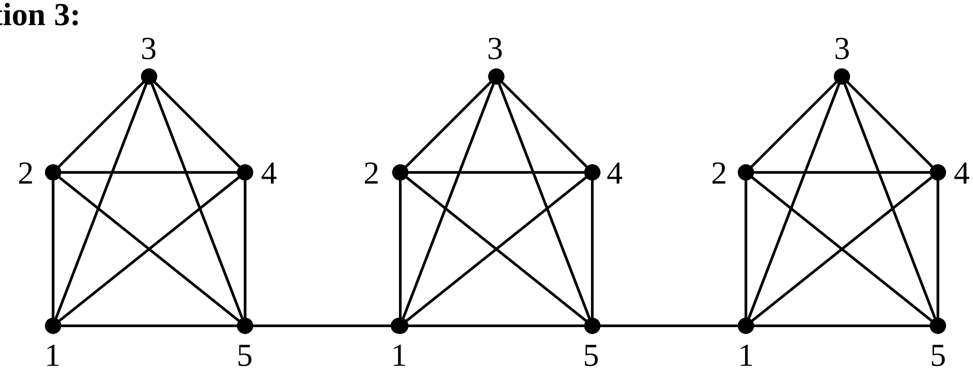

Figure 9: Barbell B(3, 5) - graph

Illustration 4:

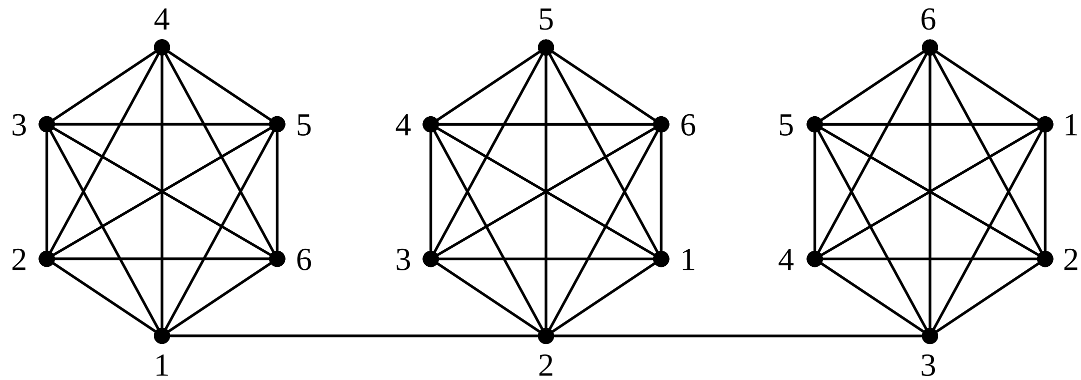

Figure 10: Barbell B(3, 6) – graph

Theorem 4:

Drum graph D_n ($n \geq 3$) acknowledges the proper coloring and its chromatic number is 3 where n is any positive integer [4].

Proof:

Let $u_1\ u_2 \ldots u_n$ be the upper vertices of the drum graph and $v_1\ v_2 \ldots v_n$ be the lower most vertices of the drum graph and let $w_1\ w_2 \ldots w_n$ be the path vertices on this graph and u be the common vertex.

Let the coloring $f : V(G) \rightarrow \{1, 2, 3\}$, as defined as follows,

First drum graph D_3 can be colored as,

(i) $f(u) = 1$
(ii) $f(u_1) = 2$
(iii) $f(u_2) = 3$
(iv) $f(v_1) = 2$
(v) $f(v_2) = 3$
(vi) $f(w_i) = 3$, for $i = 1, 3 \ldots n-1$
(vii) $f(w_i) = 2$, for $i = 2, 4 \ldots n$

Illustration:

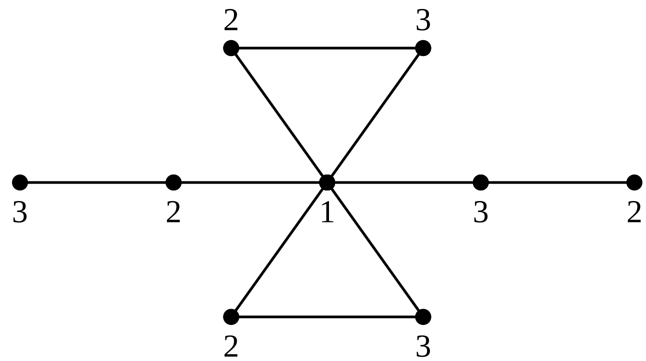

Figure 11: Drum graph D_3

Drum graph D_4 can be colored as,

(i) $f(u) = 1$

(ii) $f(u_1) = 2 = f(u_3)$

(iii) $f(u_2) = 3$

(iv) $f(v_1) = 2 = f(v_3)$

(v) $f(v_2) = 3$

(vi) $f(w_i) = 3$, for $i = 1, 3 \ldots n-1$

(vii) $f(w_i) = 2$, for $i = 2, 4 \ldots n$

Illustration:

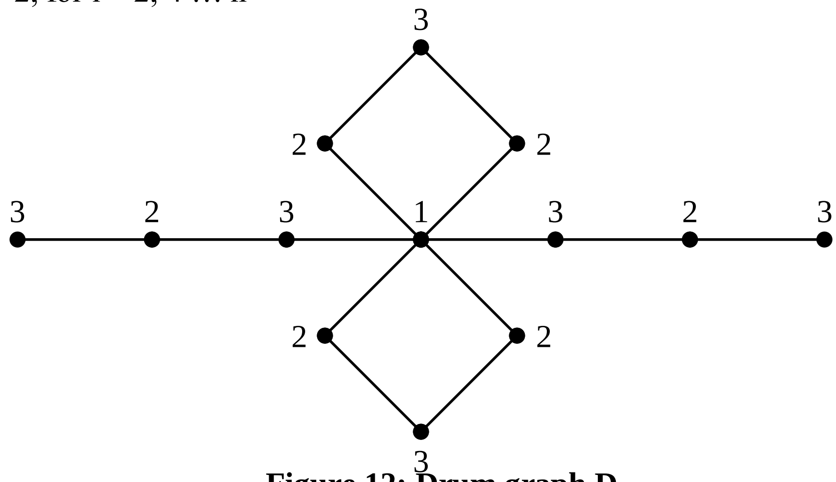

Figure 12: Drum graph D_4

Similarly drum graph D_5 can be colored as,

(i) $f(u) = 1$

(ii) $f(u_1) = 2 = f(u_3)$

(iii) $f(u_2) = 3 = f(u_4)$

(iv) $f(v_1) = 2 = f(v_3)$

(v) $f(v_2) = 3 = f(v_4)$

(vi) $f(w_i) = 3$, for $i = 1, 3 \dots n-1$

(vii) $f(w_i) = 2$, for $i = 2, 4 \dots n$

Illustration:

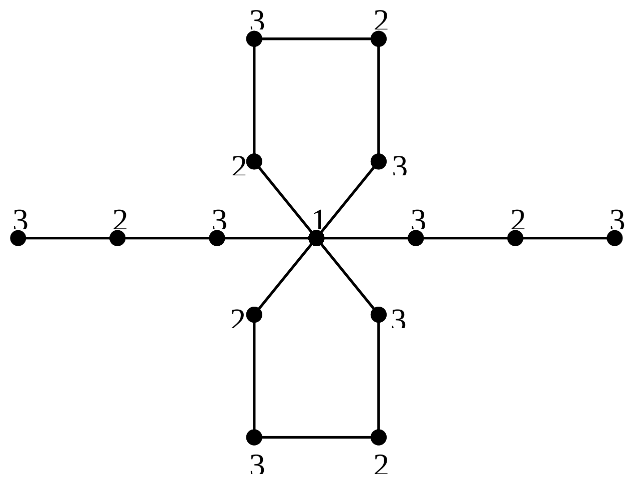

Figure 13: Drum graph D_5

Conclusion:

Thus we find the proper coloring of the above mentioned graphs. It is of interest to study, some graph labeling such as prime cordial labeling, prime harmonious labeling, for the classes of graphs like Hamiltonian graphs, Eulerian graphs, Peterson graphs etc.,

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