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STUDY OF MOVEMENT OF TWO-DIMENSIONAL CONTAMINANTS IN UNSATURATED POROUS MEDIA WITH UNIFORM FLOW

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Abstract

Contaminants released at the surface of the earth pass through the soil and contaminate the underlying ground water. Here we obtain two-dimensional mathematical model for movement of radio active contaminants through unsaturated porous media with uniform flow. The advection dispersion equation is solved subjected to the given boundary conditions. The analytical solutions of the non-linear partial differential equations are obtained using integral transform techniques in terms of error functions.

Keywords: Ground water, contaminants, unsaturated, porous media, uniform flow

Introduction

The unsaturated zone controls the transmission of contaminants and aquifer recharging water. It is a place where wastes are collected to isolate them from significant exchange with other environmental components. The flow processes that occur in the unsaturated zone contribute to a wide variety of hydrologic processes. The texture of the soil and travel distance influences the dispersion process significantly.

Many studies have been carried out to investigate solute transport processes in the saturated zone (Fitch & Jia, [2], Chiogna etal, [4]). Hongtao wang [3] obtained a library of analytical solution for contaminant transport in uniform flow in porous media with first order decay, linear sorption and zero order production. A computer code was developed to calculate the solutions. Mritunjay kumar Singh etal [5] obtained analytical solution to predict the contaminant concentration with presence and absence of pollution source in finite aquifer subject to constant point source using Laplace transformation technique. Yadav and Lav Kush Kumar [8] studied analytical solution of spatially dependent solution transport in 1-dimenional semi-infinite homogeneous porous domain. To solve the advection dispersion equation, Laplace transform technique is used. The effects of spatial dependence on the solute concentration dispersion of various physical parameters are explained with the help of graphs. Dilip kumar Jaiswal, R R Yadav [6] obtained an analytical solutions of one-dimensional advection-diffusion equation in a finite domain for two sets of pulse type boundary condition using Laplace transform. The solutions are graphically illustrated and compared solution distribution for finite and semi-infinite domain. Teodrose Atnafu Abgaze and Sharma [7] studied the behavior of breakthrough curves in mixed heterogeneous soil column experiments. Advective dispersive transport equations were used for solute transport through mobile–immobile porous medium. A hybrid finite volume method is used to solve the governing equations for solute concentration in mobile region.

The important purpose of the study is to develop a mathematical model for the two-dimensional flow of pollutants in unsaturated porous media with the uniform flow using isotopes. The advection dispersion equation is used to develop the model with suitable initial and boundary conditions to find the solution and compare it with the other solution. The magnitude of dispersion depends on the particle size distribution and flow parameters. The average pore velocity has to be considered only for the vertically downward direction.

In the present work, we have considered D_L, the longitudinal dispersion coefficient and D_T, the transverse dispersion coefficients which are perpendicular to flow directions of the fluid. The value of the D_T is more complicated to obtain than the value of D_L, because the concentration distribution are required to be measured in vertical to the flow direction. Some studies have been performed to get the transverse dispersion coefficient on the velocity of unpolluted water and porous media with the distribution of various particle sizes to find out the values of longitudinal and transverse dispersion coefficient.

Mathematical Formulation

Let us consider the two-dimensional dispersion with the one-dimensional steady-state flow is of the form

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial z^2} - v \frac{\partial C}{\partial z} + D_T \frac{\partial^2 C}{\partial x^2} - \lambda C \qquad (1)$$

where, C is the concentrationof the solution, D_L and D_T are longitudinal and transversion coefficient, t is time, v is the average pore water velocity, λ is radioactive decay constant, z, and x are the particular points along the cartesian coordinate axes which are perpendicular and parallel to the direction of groundwater flow.

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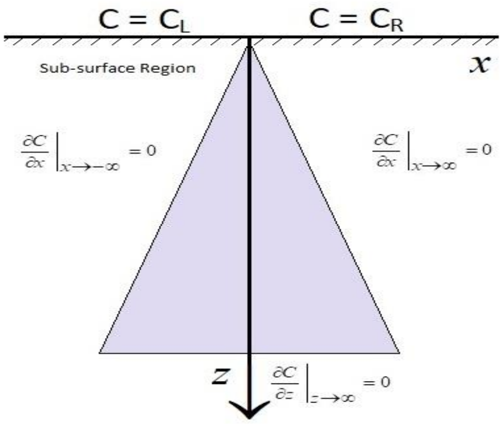

Fig.1 Physical layout of two-dimensional transport of contaminants in unsaturated porous media

For groundwater flow, a linear relation between the dispersion coefficient and seepage velocity are generally adopted. If the aquifer is isotropic dispersion coefficient can be characterized by a transverse and longitudinal coefficient (Fig.1).

For this problem the initial and boundary conditions are as follows:

$$C(x,z,0) = f(x), 0 < z < \infty, -\infty < x < \infty \tag{2a}$$

$$\left.\frac{\partial C}{\partial z}\right|_{z\to\infty} = 0, \ -\infty < x < \infty, \ t > 0 \tag{2b}$$

$$\left.\begin{aligned} C(0,x,t) &= g(x) = C_L, x < 0, t > 0 \\ C(0,x,t) &= g(x) = \frac{C_L + C_R}{2}, x = 0, t > 0 \\ C(0,x,t) &= g(x) = C_R, x > 0, t > 0 \end{aligned}\right\} \tag{2c}$$

$$\left.\frac{\partial C}{\partial x}\right|_{x\to\pm\infty} = 0, \ 0 < z < \infty, \ t > 0 \tag{2d}$$

The solution of equation (1) is obtained by using Fourier transformation for x variable and the Laplace transformation for t variable. The solution is the combination of a semi-infinite plane for steady-state conditions considering the one-dimensional advection-dispersion model and one-dimensional advection and two-dimensional dispersion model. Applying the Laplace transform

$$L\{C(x,z,t)\} = \int_0^\infty e^{-pt} C(x,z,t)\,dt = C'(x,z,p) \tag{3}$$

$$L\left[\frac{\partial C}{\partial t}\right] = pC'(x,z,p) - C(x,z,0) \tag{4}$$

equation (1) transforms to

$$pC'(x,z,p) - C(x,z,0) = D_L \frac{\partial^2 C}{\partial z^2} - v\frac{\partial C}{\partial z} + D_T \frac{\partial^2 C}{\partial x^2} - \lambda C \tag{5}$$

And boundary conditions reduces to

$$\left.\frac{\partial C'}{\partial z}\right|_{z\to\infty} = 0 \tag{6a}$$

$$C'(0,x,p) = \frac{g}{p} \tag{6b}$$

$$\left.\frac{\partial C'}{\partial x}\right|_{x\to\pm\infty} = 0 \tag{6c}$$

For the infinite x-domain the Fourier transform is applied. The Fourier transform is given by

$$F\left[C'(z,x,p)\right] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\alpha x} C'(z,x,p)\,dx = C'(z,\alpha,p) \tag{7}$$

$$F\left[\frac{\partial^2 C'}{\partial x^2}\right] = -\alpha^2 C' \tag{8}$$

The Fourier transforms of equation (5) becomes

$$D_L \frac{d^2\bar{C}'}{dz^2} - v\frac{d\bar{C}'}{dz} - (D_T\alpha^2 + p + \lambda)\bar{C}' + \bar{f} = 0 \tag{9}$$

And the reduced boundary conditions are

$$\left.\begin{aligned} &\left.\frac{d\bar{C}'}{dz}\right|_{z\to\infty} = 0 \\ &\bar{C}'(0,x.p) = \frac{\bar{g}}{p} \\ &\left.\frac{d\bar{C}'}{dx}\right|_{x\to\pm\infty} = 0 \end{aligned}\right\} \tag{10}$$

The Fourier transformation of f and g are $\bar{f}$ and $\bar{g}$.

Equation (9) subjected to the boundary condition (10) is of the form

$$\bar{C}'(z,\alpha,p) = A\left[exp(R_1 z)\right] + B\left[exp(-R_2 z)\right] + \frac{\bar{f}}{D_T\alpha^2 + \lambda + p} \tag{11}$$

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Where

$$R_1 = \frac{v}{2D_L} + \frac{\sqrt{v^2 + 4D_L\left(D_T\alpha^2 + \lambda + p\right)}}{2D_L} \tag{12a}$$

$$R_2 = \frac{v}{2D_L} - \frac{\sqrt{v^2 + 4D_L\left(D_T\alpha^2 + \lambda + p\right)}}{2D_L} \tag{12b}$$

It follows from equation (10) that

$$A = 0 \quad \text{and} \quad B = \frac{\bar{g}}{p} - \frac{\bar{f}}{D_T\alpha^2 + \lambda + p} \tag{13}$$

Substituting (6.13) in (6.11) we get

$$\bar{C}'(z,\alpha,p) = \left[\frac{\bar{g}}{p} - \frac{\bar{f}}{D_T\alpha^2 + \lambda + p}\right] exp\,(R_2 z) + \frac{\bar{g}}{p} + \frac{\bar{f}}{D_T\alpha^2 + \lambda + p} \tag{14}$$

The RHS of equation (14) has been divided into the sum of three terms, the first integral term reduces to

$$\bar{C}'(z,\alpha,p) = L^{-1}\left[\frac{\bar{g}}{p} exp\,(R_2 z)\right] \tag{15}$$

Let $h(t)$ and $k(t)$ be two functions and its Laplace transformations are $h'(p)$ and $k'(p)$ then the convolution of these integrals is

$$L^{-1}\left[h'(p).k'(p)\right] = h * k$$

$$L^{-1}\left\{h'(p)k'(p)\right\} = \int_0^t h(\tau)k(t-\tau)\,d\tau = \int_0^t h(t-\tau)k(\tau)\,d\tau \tag{16}$$

Here τ is a variable. The difference in two functions in the equation (15) is of the form

$$h'(p) = \frac{1}{p}, k(t) = exp\left[-\frac{z}{\sqrt{D_L}}\left(\frac{v^2}{4D_L} + D_T\alpha^2 + \lambda + p\right)^{1/2}\right] \tag{17}$$

The values of $h(t)$ and $k(t)$ are calculated by using shift property and Laplace transformation and is given by

$$h(t)=1,\ k(t)=\frac{z}{t\sqrt{4\pi D_L}}exp\left[-\left(\frac{\nu^2}{4D_L}+D_T\alpha^2+\lambda\right)t-\frac{z^2}{4D_Lt}\right] \tag{18}$$

Substituting equation (18) in equation (16) and subsequently into equation (15) reduces to

$$\bar{C}'(z,\alpha,p)=\frac{\bar{g}\,z}{\sqrt{4\pi D_L}}\int_0^t \tau^{\frac{3}{2}}\,exp\left(\frac{-(z-\nu\tau)^2}{4D_L\tau}\right)exp\left[(D_T\alpha^2+\lambda)\tau\right]d\tau \tag{19}$$

By using Laplace transforms table, equation (19) can be transformed in the form of the complementary error function. Now applying the complementary error function to the expression (19)

$$\bar{C}_1(z,\alpha,t)=\frac{\bar{g}}{2}exp\left(\frac{\nu z}{2D_L}\right)\left\{exp\left[\frac{z}{\sqrt{D_L}}\left(\frac{\nu^2}{4D_L}+D_T\alpha^2+\lambda\right)^{1/2}\right]*\right.$$

$$erfc\left[\frac{z}{\sqrt{4D_LT}}+\sqrt{\frac{\nu^2t}{4D_L}+D_T\alpha^2t+\lambda t}\right]\Bigg\}+\left\{exp\left[\frac{-z}{\sqrt{D_L}}\left(\frac{\nu^2}{4D_L}+D_T\alpha^2+\lambda\right)^{1/2}\right]*\right.$$

$$\left. erfc\left[\frac{z}{\sqrt{4D_LT}}-\sqrt{\frac{\nu^2t}{4D_L}+D_T\alpha^2t+\lambda t}\right]\right\} \tag{20}$$

where $erfc(z)=1-erf\,(z)=\frac{2}{\sqrt{\pi}}\int_z^{\infty}e^{-\eta^2}d\eta,\ erf\,(z)=\frac{2}{\sqrt{\pi}}\int_0^z e^{-\eta^2}d\eta$

Applying the inverse Fourier transform to the second term $\bar{C}_2(z,\alpha,p)$ of RHS of the equation (14), we have

$$\bar{C}_2(z,\alpha,t)=L^{-1}\left[-\frac{\bar{f}}{D_T\alpha^2+p+\lambda}exp(Rz)\right] \tag{21}$$

$$\bar{C}_2(z,\alpha,t)=\frac{\bar{f}}{2}exp\left[-\left(D_T\alpha^2+\lambda\right)t\right]\left\{erfc\left(\frac{z-\nu t}{\sqrt{4D_Lt}}\right)+exp\left(\frac{\nu z}{D_L}\right)\cdot erfc\left(\frac{z+\nu t}{\sqrt{4D_Lt}}\right)\right\} \tag{22}$$

Now applying inverse Laplace transform to the third term on the RHS of equation (14) using Oberhettinger and Baddi [1] we get

$$\bar{C}_3(z,\alpha,t)=L^{-1}\left[\frac{\bar{f}}{D_T\alpha^2+p+\lambda}\right]=\bar{f}\cdot exp\left[-\left(D_T\alpha^2+\lambda\right)t\right] \tag{23}$$

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The required equation $\bar{C}(z,\alpha,t)$ can be written as

$$\bar{C}(z,\alpha,t) = \bar{C}_1(z,\alpha,t) + \bar{C}_2(z,\alpha,t) + \bar{C}_3(z,\alpha,t) \tag{24}$$

The inverse Fourier transform $\bar{C}(z,\alpha,t)$ is

$$C(z,x,t) = F^{-1}\left[\bar{C}(z,x,t)\right] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} exp\left(-i\alpha x\right) \bar{C}\left(z,\alpha,t\right) d\alpha \tag{25}$$

The Fourier inversion of the first term of the equation (24) can be written as

$$F^{-1}\left[C_1\left(z,x,t\right)\right] = \frac{1}{\sqrt{2\pi}} \left\{ \int_{-\infty}^{\infty} exp\left(-i\alpha x\right) \frac{\bar{g}z}{\sqrt{4\pi D_L}} \cdot \int_0^t \tau^{-\frac{3}{2}} exp\left(\frac{(z-v_1\tau)^2}{4D_L\tau}\right) exp\left[-\left(D_T\alpha^2+\lambda\right)\tau\right] \right\} d\tau d\alpha$$

$$F^{-1}\left[C_1\left(z,x,t\right)\right] = \frac{z}{\sqrt{4\pi D_L}} \int_0^t \tau^{-\frac{3}{2}} exp\left(-\frac{(z-v_1\tau)^2}{4D_L\tau}\right) *$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} exp\left(-i\alpha x\right) \left\{\bar{g}\ exp\left[-\left(D_T\alpha^2+\lambda\right)\tau\right]\right\} d\alpha d\tau \tag{26}$$

The convolution $h*k$ is to determine the convolution of the product of two functions $\bar{h}$ and $\bar{k}$ given by

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{h}\left(\alpha\right)\bar{k}\left(\alpha\right) exp\left(-i\alpha x\right) d\alpha = h*k = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} h\left(x-\gamma\right)k\left(\gamma\right)d\gamma \tag{27}$$

Where γ is the dummy variable, after verifying the last term for $C_1(z,x,t)$ in equation (26), using the convolution integral for the other two functions

$$\bar{h}\left(\alpha\right) = \bar{g}, \bar{k}\left(\alpha\right) = exp\left[-\left(D_T\alpha^2+\lambda\right)\tau\right] \tag{28}$$

$$F^{-1}\left\{exp\left[-\left(D_T\alpha^2+\lambda\right)\tau\right]\right\} = \frac{1}{\sqrt{2\left(D_T+\lambda\right)\tau}} exp\left[-\frac{x^2}{4\left(D_T+\lambda\right)\tau}\right] \tag{29}$$

The resultant $h(x)$ and $k(x)$ are

$$\left.\begin{aligned} h(x) &= g(x) = C_L, & x<0 \\ h(x) &= g(x) = \frac{(C_L+C_R)}{2}, & x=0 \\ h(x) &= g(x) = C_{R}, & x>0 \end{aligned}\right] \tag{30}$$

and

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$$k(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}k(\alpha)exp(-i\alpha x)d\alpha=\frac{1}{\sqrt{2\left(D_T+\lambda\right)\tau}}exp\left[-\frac{x^2}{4\left(D_T+\lambda\right)\tau}\right] \tag{31}$$

Equation (27) is of the form

$$h*k=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{g}{\sqrt{2(D_T+\lambda)\tau}}exp\left[-\frac{(x-\gamma)^2}{4(D_T+\lambda)\tau}\right]d\gamma \tag{32}$$

The above expression can be evaluated by taking

$$\rho=\frac{\gamma-x}{\sqrt{4(D_T+\lambda)\tau}}$$

The conditions given in equation (30) and the complementary error function gives

$$h*k=\frac{C_L}{2}erfc\left[\frac{x}{4(D_T+\lambda)\tau}\right]+\frac{C_R}{2}erfc\left[\frac{x}{4(D_T+\lambda)\tau}\right] \tag{33}$$

The result is then substituted in (30) to get an expression for $C_1(z,x,t)$, is

$$C_1(z,x,t)=\int_0^t h*k=\frac{z}{\sqrt{4\pi D_L}}\int_0^t \tau^{-\frac{3}{2}}exp\left(-\frac{(z-\nu\tau)^2}{4D_L\tau}\right)*$$

$$\left\{\frac{C_L}{2}erfc\left[\frac{x}{4(D_T+\lambda)\tau}\right]+\frac{C_R}{2}erfc\left[\frac{x}{4(D_T+\lambda)\tau}\right]\right\}d\tau \tag{34}$$

The inverse of the second term of the equation (26), $C_2(z,x,t)$ is

$$C_2(z,\alpha,t)=-\frac{1}{2}\left\{erfc\left(\frac{z-\gamma t}{\sqrt{4D_Lt}}\right)+exp\left(\frac{\gamma z}{D_L}\right)\cdot erfc\left(\frac{z+\gamma t}{\sqrt{4D_Lt}}\right)\right\}*F^{-1}\left[\bar{f}\,exp\left[-\left(D_T+\lambda\right)t\right]\right] \tag{35}$$

Applying the convolution theorem, the inverse Fourier transform in equation (35) is given by,

$$\bar{h}(\alpha)=\bar{f},\bar{k}(\alpha)=exp\left[-\left(D_T\alpha^2+\lambda\right)\tau\right] \tag{36}$$

Let us assume that the initial concentration C_i is constant and equation (29) is utilized to find $k(x)$ can be written as

$$h(x)=C_i,k(x)=\frac{1}{\sqrt{2(D_T+\lambda)t}}exp\left[-\frac{x^2}{4(D_T+\lambda)t}\right] \tag{37}$$

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$$F^{-1}\left[\bar{f}\,exp\left[-\left(D_T\alpha^2+\lambda\right)t\right]\right]=h*k=\frac{C_i}{\sqrt{4\pi\left(D_T+\lambda\right)t}}\int_{-\infty}^{\infty}exp\left[-\frac{(x-\gamma)^2}{\sqrt{4\pi\left(D_T+\lambda\right)t}}\right]d\gamma \qquad (38)$$

Substitution of equation (31) into equation (28), $C_2(z,x,t)$ reduces to

$$C_2(z,x,t)=-\frac{C_i}{2}\left\{erfc\left(\frac{z-\gamma t}{\sqrt{4D_L t}}\right)+exp\left(\frac{\gamma z}{D_L}\right)\cdot erfc\left(\frac{z+\gamma t}{\sqrt{4D_L t}}\right)\right\} \qquad (39)$$

By taking the inverse Fourier transform of the last term of equation (24), $C_3(z,x,t)$ is

$$C_3(z,x,t)=F^{-1}\left[\bar{f}\cdot exp\left[-\left(D_T\alpha^2+\lambda\right)t\right]\right]=C_i$$

$$C_3\left(z,x,t\right)=\frac{C_i}{\sqrt{4\pi(D_T+\lambda)t}}\int_{-\infty}^{\infty}exp\left[-\frac{x-\gamma}{\sqrt{4\pi(D_T+\lambda)t}}\right]d\gamma \qquad (40)$$

Substitution of $C_1(z,x,t)$, $C_2(z,x,t)$ and $C_3(z,x,t)$ in equation (24) gives

$$C\left(z,x,t\right)=\frac{z}{\sqrt{4\pi D_L}}\int_0^t\tau^{-\frac{3}{2}}\left\{\frac{C_L}{2}erfc\left[\frac{x}{\sqrt{4(D_T+\lambda)\tau}}\right]+\frac{C_R}{2}erfc\left[\frac{-x}{\sqrt{4(D_T+\lambda)\tau}}\right]\right\}*$$

$$exp\left[-\frac{(z-\gamma\tau)^2}{4D_L\tau}\right]d\tau--\frac{C_i}{2}\left\{erfc\left(\frac{z-\gamma t}{\sqrt{4D_L t}}\right)+exp\left(\frac{\gamma z}{D_L}\right)\cdot erfc\left(\frac{z+\gamma t}{\sqrt{4D_L t}}\right)\right\}+C_i \qquad (41)$$

Various numerical methods can be used to evaluate Eqn. (41), among the suitable method of solution is Gauss – Chebyshev quadrature method.

Results and Discussions

The characters of the soil within the vicinity taken into consideration need to be homogeneous .The solution furnished in (41), is confirmed through evaluation with numerous numerical solutions for specific values of v, D_L, D_T, and λ. For this confirmation, we assumed that an absorbent medium with the following arbitrary transportation parameters: D_L=25cm^2/d, D_T=5cm^2/d, and v=50cm/d. All concentrations are expressed as dimensionless quantity C/C_0, with C_0 =1.

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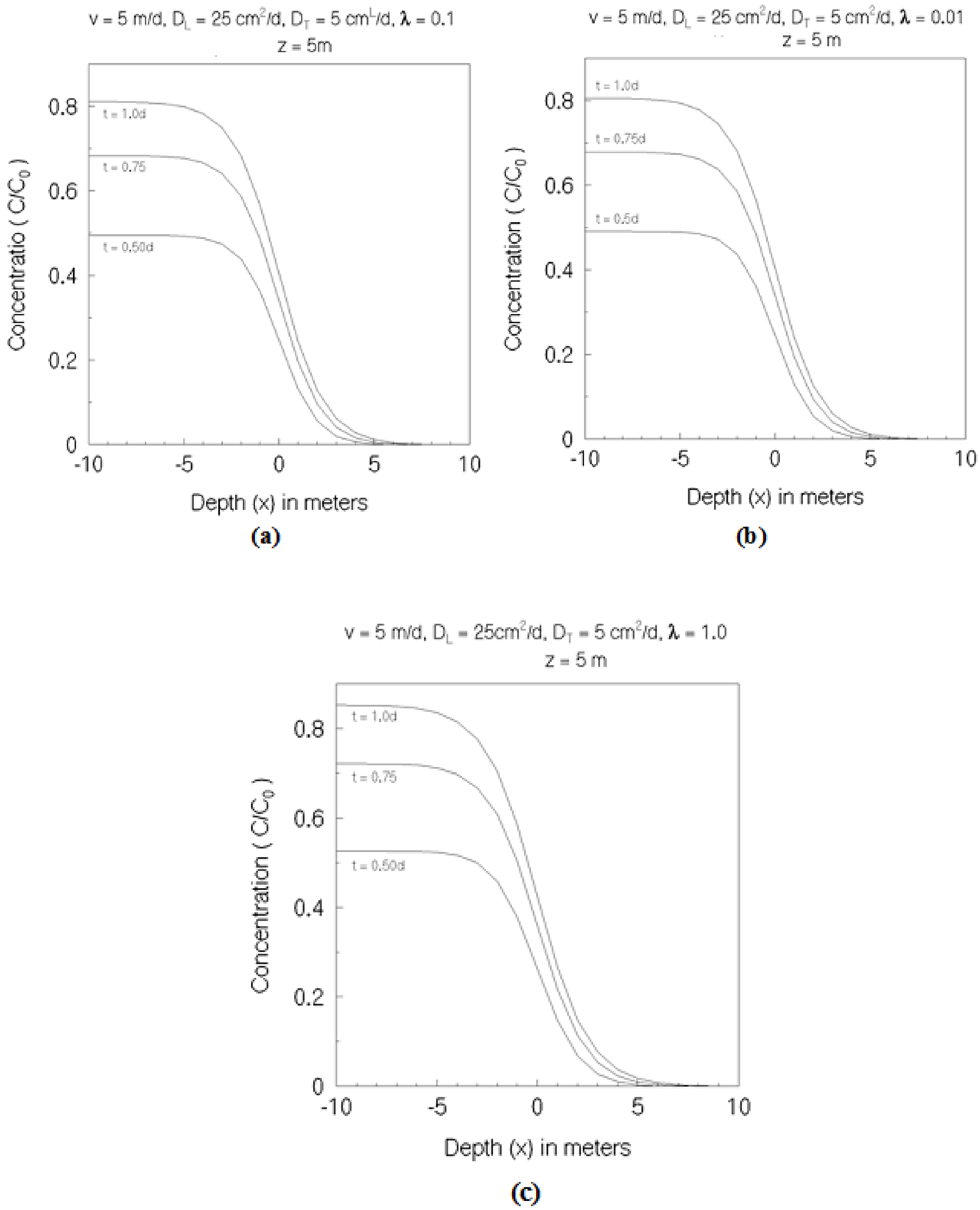

Fig.2 Break-through-curve for C /C_0 Vs depth

Fig. 2 represent the concentration profile for specific depths z , t and x and radioactive decay constant λ and time according to equation (41).The results are almost the same as the steady-state solution given above equation. Fig. 2 explain the solute movement as a result of diffusion from the hidden solute source.

Conclusion

From this problem, we are capable of seeing that an analytical answer is received for the two-dimensional advection dispersion equation for a semi-endless medium (half of the plane) using integral transforms. The main limitations of the analytical methods are that the applicability is for relatively simple problems. The geometry of the problem should be regular. The properties of the soil in the region considered must be homogeneous in the sub region.

References

1 Oberhettinger and Baddi, Tables of Laplace Transforms. Springer-Verlag (1973).
2 Fitch A and Du J, Solute transport in clay media: Effect of humic acid, *Environ. Sci. Technol.* 30, 1,(1995) 12–15.
3 Hongtao wang and Huayong W U, Analytical solutions of three-dimensional contaminant transport in uniform flow field in porous media: A library, Front. Environ. Sci. Engin.China, 3, 1, (2009)112-128.
4 Gabriele Chiogna, Christina Eberhardt, Peter Grathwohl, O laf A. Cirpka, and Massimo Rolle, Evidence of compound-dependent hydrodynamic and mechanical transverse dispersion by multitracer laboratory experiments, *Environ. Sci. Technol.* 44, 2,(2010) 688–693.
5 Mritunjay Kumar Singh, Nav Kumar Mahato and Premlata Singh, Longitudinal dispersion with constant source concentration along unsteady groundwater flow in finite aquifer: analytical solution with pulse type boundary condition, Natural Science, 3,3, (2011) 186-192.
6 Dilip kumar Jaiswal, R R Yadav, Contaminant diffusion along uniform flow velocity with pulse type input sources in finite porous medium, Int J of Appl math, electronics & computers, 2,4, (2014) 19-25.
7 Teodrose Atnafu Abgaze and P.K. Sharma, Solute transport through porous media with scale-dependent dispersion and variable mass transfer coefficient, ISH Journal of Hydraulic Engineering , 21(3), (2015) 298-311.
8 R R Yadav and Lav Kush Kumar, One-dimensional spatially dependent solute transport in semi-infinite porous media: an analytical solution, Int J Engg Sci, 9,4, (2017) 20-27.