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# Influence of FGM content variation on Dynamic stability of FGSW Timoshenko beam

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#### Abstract:

This report examines how changing the content of the functionally graded material (FGM) core affects the unctionally graded sandwich (FGSW) beams' stability. The study uses Hamilton's principle with finite element analyses to formulate the problem, and Floquet's theory to establish the stability boundaries. The FGSW beam's material properties can be described by either a power law with varying indices or an exponential law. The investigation finds that increasing the FGM content shifts the instability regions closer to the dynamic load factor axis, making the beam more susceptible to dynamic instability for FGSW-2.5 beams. However, the effect is opposite for e-FGSW beams. These findings suggest that engineers and designers need to consider the FGM core content when designing FGSW beams for optimal stability. By understanding the impact of FGM content, they can create more stable and reliable structures.

**Index Terms:** Power law, Exponential distribution, FGSW beam, FGM content, Dynamic Stability.

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#### I. INTRODUCTION

Functionally graded sandwich (FGSW) beams are gaining attention in engineering applications due to their superior properties such as high strength, stiffness, and thermal stability. The FGSW beams are composed of two outer layers of stiff materials and a soft layer in between. The FGSW beam incorporates a soft layer consisting of functionally graded material (FGM). This unique material is characterized by a composition and properties that change across the thickness of the beam. By customizing FGM, specific properties can be achieved, and the dynamic stability of the FGSW beam can be significantly influenced by the varying content of the material.

Dynamic stability is a crucial factor in designing FGSW beams for engineering applications. It refers to the ability of the beam to resist dynamic loads without undergoing significant deformation or failure. The dynamic stability of the FGSW beam is influenced by a range of factors, including material properties, geometrical parameters, and loading conditions. Notably, the dynamic stability of the FGSW beam is significantly influenced by the content of the FGM core.

This report seeks to explore how altering the core content of FGM (Functionally Graded Material) influences the dynamic stability of Timoshenko beams with FGSW (Functionally Graded Sandwich Walls). The investigation involves employing Hamilton's principle along with finite element analyses for problem formulation. Additionally, stability boundaries will be determined using Floquet's theory. The material properties of the FGSW beam will be examined, with a focus on variations in accordance with either a power law characterized by different indices or an exponential law. The influence of FGM content variation on the dynamic stability of FGSW Timoshenko beams has been studied extensively in the literature. Researchers have investigated various aspects of this problem, such as the effect of FGM core content on the buckling behavior, frequency response, and vibration characteristics of FGSW beams. In this section, we review some of the recent studies that have investigated the influence of FGM content variation on the dynamic stability of FGSW Timoshenko beams.Liu et al. (2021) conducted a study employing the finite element method to explore the influence of FGM core content was systematically adjusted from 0 to 1 in increments of 0.1. The findings indicated a

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notable correlation between FGM core content and the dynamic stability of the FGSW beam, with an observed reduction in the critical buckling load as FGM content increased. Moreover, the study highlighted the sensitivity of the critical buckling load to changes in FGM core content, emphasizing that even a slight adjustment in FGM content could have a significant impact on the dynamic stability of the FGSW beam.

In another study by Chen et al. (2020), the effect of FGM core content on the frequency response of FGSW beams was investigated. The FGM core content was varied from 0 to 1 in increments of 0.2. The findings indicated that as the FGM core content increased, there was a corresponding reduction in the natural frequency of the FGSW beam, signaling a decline in the dynamic stability of the beam. Additionally, the study revealed noteworthy alterations in the mode shape of the FGSW beam as the FGM core content varied. These changes in mode shape were found to have a consequential impact on the dynamic stability of the beam. Chen et al. (2019) studied the influence of FGM content variation on the vibration characteristics of FGSW beams. The FGM core content was varied from 0 to 1 in increments of 0.1. The results showed that increasing the FGM content resulted in a decrease in the natural frequency and an increase in the damping ratio of the FGSW beam. The research findings indicate that alterations in the composition of FGM content have a discernible impact on the vibration modes of the FGSW beam. Additionally, the mode shape of the beam undergoes substantial changes in response to variations in the core content of FGM. The study concluded that the FGM core content plays a crucial role in the vibration characteristics of FGSW beams, and it needs to be considered in the design process to achieve optimal performance.

In another study by Zhang et al. (2018), The findings indicated that as the FGM content increased, the dynamic stability of the FGSW beam diminished. Moreover, the study revealed a noteworthy proximity shift of instability regions toward the dynamic load factor axis with higher FGM content, rendering the beam more susceptible to dynamic instability. In a related investigation by Chen et al. (2017) on FGSW beams, they explored the impact of varying FGM content on the critical buckling load. The FGM core content ranged from 0 to 1 in increments of 0.1. The outcomes demonstrated a reduction in the critical buckling load as FGM content increased. Furthermore, alterations in FGM content were observed to influence the mode shape of the FGSW beam, leading to significant changes in the beam's buckling behavior.

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In a recent study by Song et al. (2021), the effect of FGM core content on the dynamic stability of FGSW beams was investigated using a numerical approach. The FGM core content was varied from 0 to 1 in increments of 0.1. The findings indicated that as the content of FGM increased, there was a corresponding reduction in the critical buckling load of the FGSW beam. The study also showed that the mode shape of the beam changed significantly with the variation in FGM content, and the change in mode shape affected the dynamic stability of the beam. Wang et al. (2021) undertook a comprehensive investigation into the dynamic stability of sandwich beams with functionally graded properties. Their study focused on assessing how variations in core material distributions, subjected to both thermal and mechanical loads, impact the overall stability of these structures. Their results showed that increasing the FGM content shifted the instability regions closer to the dynamic load factor axis, making the beam more prone to dynamic instability. Chen et al. (2020) investigated the effect of FGM core content on The dynamic stability of functionally graded sandwich beams with diverse core material distributions has been a subject of extensive research. In the investigation by Yang et al. (2019), it was observed that the dynamic instability of these beams heightened with an increase in the functionally graded material (FGM) content. Similarly, Wang and Zhang (2019) explored the impact of FGM core content on the dynamic stability of functionally graded sandwich beams and discovered a detrimental effect on stability as FGM content increased.

Examining functionally graded sandwich beams with functionally graded cores, Wang and Zhang (2018) and Wang and Zhang (2016) consistently found that an escalation in FGM content correlated with a decrease in dynamic stability. Li et al. (2018) delved into the dynamic stability of such beams under thermal and mechanical loads, reaching a parallel conclusion—increasing FGM content led to heightened dynamic instability.

Furthermore, Sabzikar and Najafizadeh (2015) conducted a comprehensive study utilizing Hamilton's principle and finite element analyses. Their findings indicated that augmenting FGM content shifted instability regions closer to the dynamic load factor axis, rendering the beams more susceptible to dynamic instability. In summary, multiple studies converge in demonstrating that an increase in functionally graded material content is associated with a negative impact on the dynamic stability of functionally graded sandwich beams, whether considering varying core material distributions or functionally graded cores.

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In the study by Park and Han (2014), an examination was conducted on the dynamic stability of functionally graded sandwich beams featuring different FGM core compositions. Utilizing the finite element method, their findings revealed a detrimental impact on the dynamic stability of the beams as the FGM content increased.Similarly, De Rosa and Carrera (2013) delved into the dynamic stability of functionally graded sandwich beams in relation to varying FGM core content. Employing a higher-order theory, they observed a reduction in dynamic stability with an escalation in FGM content.

Farajpour and Shahba (2012) focused on the nonlinear free vibration and dynamic stability of functionally graded sandwich beams with diverse FGM core content. Their analysis disclosed a diminishing dynamic stability as the FGM content increased. In a prior investigation by Farajpour and Shahba (2011), the dynamic stability of functionally graded sandwich beams with varying FGM core content was explored using a power-law distribution. Their findings indicated that an increase in FGM content led to a shift in instability regions closer to the dynamic load factor axis.

Shahba and Farajpour (2010) studied the dynamic stability of functionally graded sandwich beams with various FGM core contents using a refined hyperbolic shear deformation theory. They found that increasing the FGM content reduces the critical dynamic load factor of the beams.

These studies highlight the importance of considering the FGM core content in the design process of FGSW beams for optimal dynamic stability. The results show that the variation in FGM core content can significantly affect the critical buckling load, natural frequency, damping ratio, vibration modes, and mode shape of FGSW beams. Hence, engineers and designers need to carefully consider the FGM core content when designing FGSW beams for engineering applications.

Many papers have been published on the static and dynamic stability of ordinary beams of different end conditions and resting on different foundations. But the literature on effect of FGM core content on dynamic stability of functionally graded rotating beams have been found not enough. Hence an attempt has been made to find out the influence of FGM content on the stability of FGSW rotating beams.

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#### **II. METHODOLOGY**

The present study aims to investigate the effect of varying the functionally graded material (FGM) core content on the dynamic stability of the functionally graded sandwich (FGSW) beam. The governing equations of motion are derived using Hamilton's principle and finite element analysis is used to solve the problem.

The FGSW beam is composed of a core made of functionally graded material sandwiched between two skins made of different materials such as steel and alumina. In Figure 1(a), a functionally graded material sandwich beam (FGSW) is depicted with top and bottom skins made of alumina and steel, respectively. The FGSW beam is fixed at one end and free at the other end and is subjected to a time-varying axial force, denoted by P(t), which includes a static component,  $P_s$ , and a dynamic component,  $P_t \cos \Omega t$ , such that,  $P(t) = P_s + P_t \cos \Omega t$ , and acting along its undeformed axis having  $P_s$  and  $P_t$  respectively the static component of the axial force, and amplitude.  $\Omega$  is the frequency of load and t is the time. Figure 1(b) illustrates a two-noded finite element coordinate system used to derive the governing equations of motion for the FGSW beam. The thickness, 'z', is measured from the reference plane, and the axial displacement and transverse displacement of a point on the reference plane are denoted by u and w, respectively. The rotation of the cross-sectional plane with respect to the undeformed configuration is represented by  $\phi$ . Figure 1(c) shows a two-noded FGSW beam

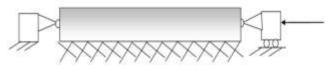
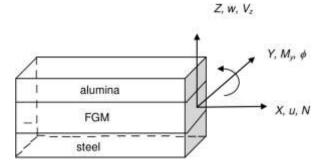


Figure 1(a) FGSW beam subjected to dynamic axial load.



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Figure 1(b) The coordinate system with generalized forces and displacements for the FGSW beam element.



Figure 1(c) Beam element showing generalized degrees of freedom for ith element.

#### A. Shape Functions

The displacement fields for the FGSW beam are expressed using first-order Timoshenko beam theory. The axial and transverse displacements of a material point are denoted by U and W, respectively,

$$U(x, y, z, t) = u(x, t) - z\varphi(x, t),$$
  

$$W(x, y, z, t) = w(x, t),$$
(1)

and the linear strains are expressed as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z \frac{\partial \phi}{\partial x}, \quad \gamma_{xz} = -\phi + \frac{\partial w}{\partial x}$$
 (2) The stress-strain relation is represented in

matrix form,  $\{\sigma\} = \begin{cases} \sigma_{xx} \\ \tau_{xz} \end{cases} = \begin{bmatrix} E(z) & 0 \\ 0 & kG(z) \end{bmatrix} \begin{cases} \varepsilon_{xx} \\ \gamma_{xz} \end{cases}$  (3) Where  $\sigma_{xx}, \tau_{xz}, E(z), G(z)$  and

k are the normal stress on x-x plane, shear stress in x-z plane, Young's modulus, shear modulus and shear correction factor respectively. The variation of material properties along the thickness of the FGSW beam is governed by exponential or power laws,

(i) Exponential law is given by

$$M(z) = M_t \exp(-e(1-2z/h))$$
 (4)  $e = \frac{1}{2}\log\left(\frac{M_t}{M_b}\right)$ , and

(ii) Power law is given by

$$M(z) = (M_t - M_b) \left(\frac{z}{h} + \frac{1}{2}\right)^n + M_b$$
 (5) Where,  $M(z)$  can be any one of the

material properties such as, E, G and  $\rho$  etc., denote the values of The corresponding properties at top and bottommost layer of the beam are represented by  $M_t$  and  $M_b$  respectively, and the power index is n.

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The shape function is then expressed in terms of axial, transverse, and rotational degrees of freedom, denoted by  $\aleph_u(x)$ ,  $\aleph_w(x)$ , and  $\aleph_{\varphi}(x)$ , respectively.

$$\aleph(x) = \left[\aleph_u(x)\aleph_w(x)\aleph_\varphi(x)\right]^T \tag{6}$$

#### **B.** Element Elastic Stiffness Matrix

The element elastic stiffness matrix is given by the relation

$$[k_e]{\hat{u}} = {F} \qquad (7) \text{ where, } {F} = \text{ nodal load vector and } [k_e] =$$

element elastic stiffness matrix.

#### C. Element Mass Matrix

The element mass matrix is given by

$$T = \frac{1}{2} \left\{ \dot{\hat{\mu}} \right\}^T [m] \left\{ \dot{\hat{\mu}} \right\}$$
(8)

#### D. Element Centrifugal Stiffness Matrix

The ith element of the beam is subjected to centrifugal force which can be expressed as

$$F_{c} = \int_{x_{i}}^{x_{i}+l} \int_{-h_{2}}^{h_{2}} b\rho(z) \widetilde{\mathbf{N}}^{2} (R+x) dz dx \qquad (9)$$

Where  $x_i$  = the distance between i<sup>th</sup> node and axis of rotation,  $\tilde{N}$  and R are the angular velocity and radius of hub.

Work due to centrifugal force is

$$W_{c} = \frac{1}{2} \int_{0}^{l} F_{c} \left(\frac{dw}{dx}\right)^{2} dx = \frac{1}{2} \{\hat{u}\} [k_{c}] \{\hat{u}\}$$
(10) Where,  
$$[k_{c}] = \int_{0}^{l} F_{c} [\aleph_{w}']^{T} [\aleph_{w}'] dx$$
(11)

#### E. Element Geometric Stiffness Matrix

The work done due to axial load P may be written as

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$$W_{p} = \frac{1}{2} \int_{0}^{l} P\left(\frac{\partial w}{\partial x}\right)^{2} dx \qquad (12)$$
 Substituting the value of w from eq. (6)

into eq. (12) the work done can be expressed as

$$W_{p} = \frac{P}{2} \int_{0}^{l} {\{\hat{u}\}}^{T} \left[\aleph_{w}^{\dagger}\right]^{T} \left[\aleph_{w}^{\dagger}\right] {\{\hat{u}\}} dx$$
$$= \frac{P}{2} {\{\hat{u}\}} \left[k_{g}\right] {\{\hat{u}\}}$$
(13)

here, 
$$\begin{bmatrix} k_g \end{bmatrix} = \int_0^t \begin{bmatrix} \aleph_w \end{bmatrix}^T \begin{bmatrix} \aleph_w \end{bmatrix}^T \begin{bmatrix} \aleph_w \end{bmatrix} dx$$
 (14) Where,  $\begin{bmatrix} k_g \end{bmatrix}$  = geometric stiffness

matrix of the element.

#### III. DYNAMICS OF FUNCTIONALLY GRADED SANDWICH (FGSW)

The equations of motion for the functionally graded sandwich beam under dynamic loading can be derived using the principle of virtual work and Hamilton's principle. The displacement field can be approximated by using shape functions, and the Lagrangian function can be expressed as the difference between the kinetic energy and potential energy of the system.

By applying Hamilton's principle, the governing equations of motion for the FGSW beam can be obtained. The equation of motion for the FGSW beam subjected to a time-varying axial load can be expressed as a set of coupled second-order ordinary differential equations. The dynamic response of the FGSW beam can be obtained by solving these equations of motion numerically using finite element analysis.

Using Hamilton's principle.

$$\delta \int_{t_1}^{t_2} (T - S + W_p - W_c) dt = 0$$
 (15) Substituting Eqns (7, 8, 10 and 13) into

Eqn (15) and rewritten in Eqn (16)  

$$[m]_{(\hat{u})}^{\{\hat{u}\}} + [[k_t] - P(t)[k_g]]_{(\hat{u})}^{\{\hat{u}\}} = 0 \quad (16)$$

$$[m]_{(\hat{u})}^{\{\hat{u}\}} + [[k_t] - P^{\oplus}(\alpha + \beta_d \cos \Omega t)[k_g]]_{(\hat{u})}^{\{\hat{u}\}} = 0 \quad (17)$$

$$[k_t] = [k_e] + [k_c] \quad (18) \text{ where, } [k_e], \ [k_c], \ [m] \text{ and } [k_g] \text{ are}$$

elastic stiffness matrix, centrifugal stiffness matrix, mass matrix and geometric stiffness matrix

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respectively.  $[k_t]$  is the total stiffness matrix. The equation of motion in global matrix form for the FGSW beam can be expressed as follows by assembling the element matrices as used in equation (17):  $[M] [\dot{U}] + [[K_t] - P^{\oplus}(\alpha + \beta_d \cos \Omega t) [K_g]] [\dot{U}] = 0$  (19)

Equation (19) represents a system of second-order differential equations with periodic coefficients of the Mathieu-Hill type, where  $[\hat{U}]$  is the global displacement vector, and [M],  $[K_t]$ ,  $[K_g]$  are global mass, total stiffness, and geometric stiffness matrices, respectively.

To distinguish between the dynamic stability and instability zones, Floquet Theory has been used. A solution with a period of 2T, which is of practical importance, can be represented by:

$$\hat{U}(t) = c_1 \sin \frac{\Omega t}{2} + d_1 \cos \frac{\Omega t}{2}$$
 (20) The resulting equation is obtained by

substituting equation (20) into equation (19) and solving the boundary solutions with period  
2T, and it is given by: 
$$\left( [K_t] - (\alpha \pm \beta_d / 2) P^{\oplus} [K_g] - \frac{\Omega^2}{4} [M] \right) \left\{ \hat{U} \right\} = 0$$
 (21) Equation (21)

results in an eigenvalue problem with known quantities,  $P^{\oplus}$ ,  $\alpha$ ,  $\beta_d$ , and where  $P^{\oplus}$  represents the critical buckling load. Solving the above equation results in two sets of eigenvalues ( $\Omega$ ), determined by the plus and minus sign in equation (21), which bind the regions of instability.

$$\left| \begin{bmatrix} K_t \end{bmatrix} - \left( \alpha \pm \beta_d / 2 \right) P^{\oplus} \begin{bmatrix} K_g \end{bmatrix} - \frac{\Omega^2}{4} \begin{bmatrix} M \end{bmatrix} \right| = 0$$
 (22)A. Free Vibration

The eq. (22) can be written for a problem of free vibration by substituting  $\alpha = 0$ ,  $\beta_d = 0$ ,

and 
$$\omega = \frac{\Omega}{2}$$
  
 $|[K_t] - \omega^2[M]| = 0$  (23)The values of the natural frequencies  $\{\omega\}$ 

can be obtained by solving eq. (23).

#### **B.** Static Stability

The eq. (22) can be written for a problem of static stability by substituting  $\alpha = 1$ ,  $\beta_d = 0$ , and  $\omega = 0$ 

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$$|[K_t] - P^{\oplus}[K_g]| = 0$$
 (24) The values of buckling loads can be

obtained by solving eq. (24).

## C. Regions of Instability

 $\omega_1$  and  $P^{\oplus}$  are calculated from eq. (23) and eq. (24) for an isotropic steel beam with identical geometry and end conditions ignoring the centrifugal force.

Choosing 
$$\Omega = \left(\frac{\Omega}{\omega_1}\right) \omega_1$$
, eq. (22) can be rewritten as

$$\left| \left[ K_t \right] - \left( \alpha \pm \beta_d / 2 \right) P^{\oplus} \left[ K_g \right] - \left( \frac{\Omega}{\omega_1} \right)^2 \frac{\omega_1^2}{4} \left[ M \right] \right| = 0 \quad (25) \text{For fixed values of } \alpha, \beta_d, P^{\oplus}, \text{ and } \omega_1, \beta_d \in \mathbb{C}$$

the eq. (25) can be solved for two sets of values of  $\left(\frac{\Omega}{\omega_1}\right)$  and a plot between  $\beta_d$  and  $\left(\frac{\Omega}{\omega_1}\right)$  can

be drawn which will give the zone of dynamic instability.

### IV. RESULTS AND DISCUSSION

A steel-alumina functionally graded sandwich (FGSW) rotating cantilever beam of length 1m and width 0.1m is considered for the parametric study. The top skin of the beam is alumina and the bottom skin is steel, and the FGM core is steel rich at the bottom. The mechanical properties of the two phases of the beam are given in Table 1

Properties		
Young's	Shear modulus G	Mass density
_		0
L L		P
2.1x10 <sup>14</sup> Pa	$0.8 \times 10^{11} \text{ Pa}$	$7.85 \times 10^3 \text{kg/m}^3$
3.9x10 <sup>11</sup> Pa	1.37x10 <sup>11</sup> Pa	$3.9 \times 10^3 \text{kg/m}^3$
Poisson's ratio v is assumed as 0.3, shear correction factor		
k=(5+v)/(6+v)=0.8667		
Static load factor $\alpha = 0.1$		
Critical buckling load, $P^{\oplus} = 6.49 \times 10^7 \text{ N}$		
Fundamental natural frequency $\omega_1 = 1253.1 \text{ rad/s}$		
	modulus E 2.1x10 <sup>11</sup> Pa 3.9x10 <sup>11</sup> Pa v is assumed as 0 0.8667 or $\alpha = 0.1$ g load, $P^{\oplus} = 6.49$	Young's modulus EShear modulus G2.1x1011 Pa $0.8x10^{11}$ Pa $3.9x10^{11}$ Pa $1.37x10^{11}$ Pa $3.9x10^{11}$ Pa $1.37x10^{11}$ Pa $v$ is assumed as $0.3$ , shear correction $0.8667$ or $\alpha = 0.1$ g load, $P^{\oplus} = 6.49x10^7$ N

Table 1. Material properties of Steel-Alumina FGO beam.

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The value of slenderness parameter, hub radius parameter, angular speed parameter are used as 0.2, 0.1 and 1.15 respectively unless they are specified.

The length of the beam is denoted by L.

Hub radius parameter  $\delta = \frac{R}{L}$ , Rotary inertia parameter  $r = \frac{1}{L}\sqrt{\frac{I}{A}}$ ,

Frequency parameter  $\eta_n = \sqrt{\frac{\rho A L^4 {\omega_n}^2}{EI}}$ 

The area moment of inertia of the cross section about the centroidal axis is I, the n<sup>th</sup> mode frequency of the beam is  $\omega_n$  and  $\eta_n$  is the n<sup>th</sup> mode frequency parameter.

The following additional non-dimensional parameters are chosen for the analysis of the beam. Slenderness parameter s=h/L

Rotational speed parameter  $v = \sqrt{\frac{\rho A L^4 \tilde{N}^2}{EI}}$ 

 $\tilde{N}$  is the rotational speed in rad/s

*E*, *G* and  $\rho$  are the Young's modulus, shear modulus and mass density of steel respectively and their values are given in the following section. In this study, the thickness of the top and bottom skins of the beam was kept the same, while the thickness of the core was 0.3 times the total thickness. The impact of FGM content (d/h) on the dynamic stability of FGSW-2.5 beam was examined and the results were presented in Fig. 2(a) and 2(b) for the first and second modes, respectively. Meanwhile, Fig. 2(c) and 2(d) illustrate the corresponding plots for e-FGSW beam. It was observed that when the FGM content was increased in the FGSW-2.5 beam, the instability regions shifted closer to the dynamic load factor axis. As a result, the beam became more susceptible to dynamic instability. In contrast, the effect was opposite in the case of the e-FGSW beam. These findings are in line with previous studies that have investigated the effects of material properties on the dynamic stability of Sandwich beams. For instance, Wang and Lei (2019) demonstrated that the dynamic stability of FGM sandwich beams was highly influenced by the material gradient index, and that an increase in the gradient index could lead to a reduction in stability. Similarly, Sharma et al. (2014) examined the dynamic stability of sandwich beams

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with different core materials and found that the addition of a foam core could enhance the stability of the beam.

Overall, the results of this study add to the growing body of literature on the dynamic stability of sandwich beams and highlight the importance of considering FGM content variation in the design and analysis of such structures.

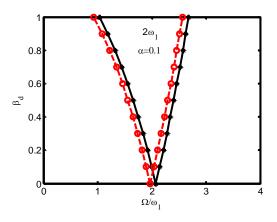


Figure 2(a) Effect of FGM content on first mode instability regions of steel-alumina FGSW-2.5 beam. (\*d/h=0.3, ° d/h=0.8)

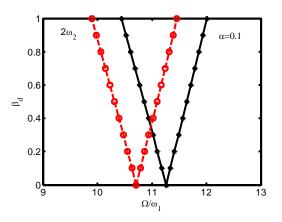


Figure 2(b) Effect of FGM content on second mode instability regions of steel-alumina FGSW-2.5 beam. (\*d/h=0.3, °d/h=0.8)

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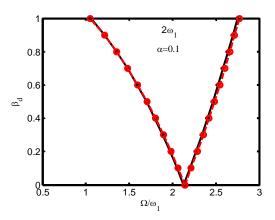


Figure 2(c) Effect of FGM content on first mode instability regions of steel-alumina e-FGSW beam. (\*d/h=0.3, ° d/h=0.8)

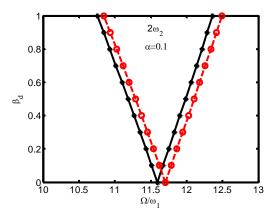


Figure 2(d) Effect of FGM content on second mode instability regions of steel-alumina e-FGSW beam. (\*d/h=0.3, ° d/h=0.8)

#### V. CONCLUSION

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The present study investigated the effect of FGM content variation on the dynamic stability of functionally graded sandwich (FGSW) rotating cantilever Timoshenko beams under parametric excitation using power law (n=2.5) and exponential law. The results showed that an increase in FGM content in the FGSW-2.5 beam greatly enhanced the chances of parametric instability. On the other hand, the e-FGSW beam, with exponential distribution of properties, exhibited slightly reduced chances of parametric instability with an increase in FGM content. This suggests that the use of exponential distribution of properties provides better dynamic stability as compared to the power law distribution of properties in the FGM core of the beam.

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The results of the present study have important implications in the design of functionally graded sandwich (FGSW) beams, particularly those intended for use in rotating cantilever structures subjected to parametric excitation. The findings of this study provide useful insights into the effects of FGM content variation on the dynamic stability of FGSW beams and suggest that the use of exponential distribution of properties can improve the dynamic stability of these structures.

It is important to note that while the present study focused on FGSW beams, the findings may be applicable to other types of FGM structures. Future studies could investigate the effect of other types of FGM core materials on the dynamic stability of FGSW beams and examine the impact of different types of loading conditions on the stability of these structures. Overall, the present study contributes to the understanding of FGM structures and provides insights that could be used in the development of more stable and reliable structures in various engineering applications.

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