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A Study on Cubic Difference Prime Labeling Ladder Graphs

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Abstract:

A primary labelling of a graph with n vertices is a labelling of its vertices in a manner that the labels of each two adjacent vertices are relatively primitive with unique numbers of $\{1,2,\ldots,n\}$. T. Varkey suspected of prime labelling in the ladder graphs. That's what we prove. in this conjecture.

Keywords: Ladder, Coprime Graph, Prime Labelling, Circle Graph , Middle Graph.

Introduction:

Let G be a basic n vertical graph. A prime labelling of G is a label of its vertices which has different entries from $\{1,2,\ldots,n\}$, in order to provide substantially prime labelling for any two neighboring vertices. The coprime integer diagram (see [14, Section 7.4]) is established as a vertex when two vertices are near, provided that they are relatively prime. Therefore, for the n -vertex graph, the prime labelling of $\{1,2,\ldots,n\}$ is equal to the coprime graph of the inducted subgraph. Ahlswede and Khachatrian Erd Ahls Erdös, Sárközy, and Szemerédi have investigated several characteristics of coprime of integer, including its subgraphs.

Sárközy and Sárközy See also an investigation of the known findings on this topic.

Entringer was responsible for the concept of prime labelling and was presented in Entringer assumed that all trees had a prime etiquette about 1980. There has been little development in this field until lately when it has been shown in 133 that there is an integer, $n-0$. All trees of this kind have a prime labelling with at least $n-0$. In addition, many graph classes have a prime labelling, In addition, see and the aforementioned papers on coprime graphs for more details. One of the graph classes for which the existence of prime labeling is unknown are ladders. Letting P_n denote the path graph on n vertices, the Cartesian product $P_n \times P_2$ is called the n -ladder groph. This article is intended to demonstrate the following theorem.

Theorem 1.1. Ladders have a prime labeling.

This result confirms a conjecture of Partial results on this conjecture have been reported in | 4,12,16, [17, [20].

1.2 Proof of Theorem

In this section we give a proof for Theorem II. Let's say that a labeled square fulfils the cross condition if the labels of its vertices satisfy

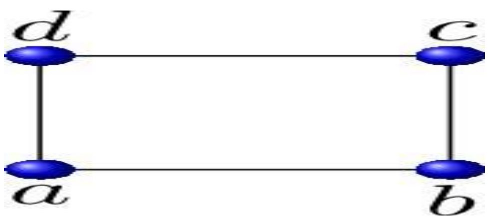

Fig: 1;1

$$\gcd(a,d)=\gcd(b,c)=\gcd(a,c)=\text{god}(b,d)=1$$

If we have a labeled ladder every square of which fulfils the cross condition, then we can alternately flip the labels of vertical edges, and as a result we obtain a ladder with a prime

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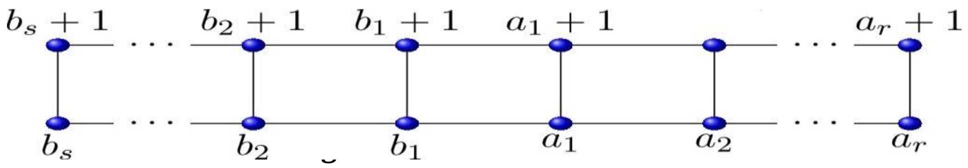
labeling. Therefore, it suffices to prove that ladders have a labeling for which every square fulfils the cross condition.

We fix an integer $n \geq 4$, and show that the n -ladder has such a labeling. The assertion for $n \leq 3$ can be verified easily.

For any positive integer k we set

$$a_k := 6k - 3 \text{ and } b_k := 6k - 1$$

We start with the following labeled $(r+s)$ -ladder



where $r = \lfloor (n+1)/3 \rfloor$ and $s = \lfloor n/3 \rfloor$. Our goal is to complete this to a labeled n -ladder in which every square fulfils the cross condition.

For positive integers ℓ satisfying

$$(5 | \ell + 3, 7 | \ell + 1) \text{ or } (5 | \ell, 7 | \ell + 3)$$

we define $c_\ell := a_\ell - 2$ and we set

$$C := \{c_\ell : a_\ell - 2 \leq 2n - 1\}.$$

(Note that the smallest ℓ satisfying (1) is $\ell = 25$, and so the smallest n for which C is nonempty is 73.) If c_ℓ exists, we 'insert' it between a_ℓ and $a_{\ell+1}$ as follows.

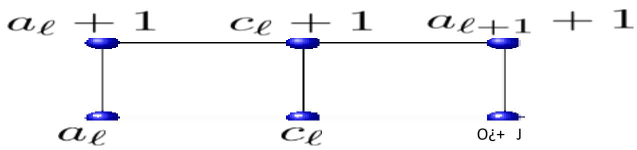


Fig:1:3

In what follows by inserting an integer between two other ones, we mean the same thing as what we did above for c_ℓ and $a_\ell, a_{\ell+1}$. Note that here it is possible that $a_{\ell+1}$ be larger than $2n$ and may not be presented in our labeling. Now, let

$$D := \{6\ell + 1 : 13 \leq 6\ell + 1 \leq 2n - 1\} \setminus C.$$

If $6\ell + 1 \in D$ and ℓ satisfies

$$5 \mid \ell + 1 \text{ or } (5 \mid \ell + 3, 7 \nmid \ell + 1) \text{ or } (7 \mid \ell + 3, 5 \nmid \ell, 5 \nmid \ell + 2)$$

then we insert $6\ell + 1$ between a_ℓ and $a_{\ell+1}$:

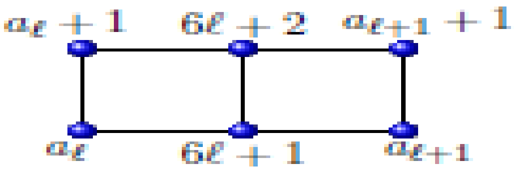


Fig: 1:4

If $6\ell + 1 \in D$ and ℓ does not satisfy ((2)), then we insert $6\ell + 1$ between b_ℓ and $b_{\ell+1}$. Note that if ℓ satisfies (12), it does not satisfy ((I) and so no integer from C is already inserted between a_ℓ and $a_{\ell+1}$. Finally we insert 7 between a_1 and b_1. By now, we have a labeled $(n - 1)$ -ladder with labels $3, \ldots, 2n$. We will insert 1 in an appropriate place later on.

Before going further with the proof, we present an example to clarify the strategy outlined above.

Example 1.2. Let $n = 9$ and then $r = s = 3$. We first build the $(r + s)$ -ladder with the following labeling:

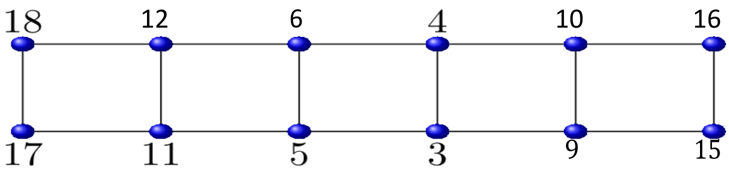


Fig: 1:5

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In this 6-ladder, the middle and far-right labeled squares do not fulfil the croes condition. Here,

In this 6-ladder, the middle and far-right labeled squares do not fulfil the cross condition. Here, the set C is empty, and $D = \{13\}$. So we insert 13 between $a_2 = 9$ and $a_3 = 15$ yielding the following labeling of the 7 -ladder:

This edge addition fixes the far-right labeled square of the former 6 -ladder so that the cross condition now exists in the two newly formed labeled squares. Lastly, we place a vertical edge with the label 7 between $a_1 = 3$ and $b_1 = 5$ to create the following 8 -ladder:

All the squares of the 8-ladder fulfil the cross condition. Finally, we insert 1 between any two vertices to obtain a 9 -ladder with a prime labeling.

Remark 3. In what follows we frequently make use of the facts that for integers u, v, h,

$$\gcd(u, v) = \gcd(u, v + uh)$$

Now, back to the proof, we list all the possible squares of our labeled ladder and verify if they fulfil the cross condition.

Fig: Clearly fulfils the cross condition.

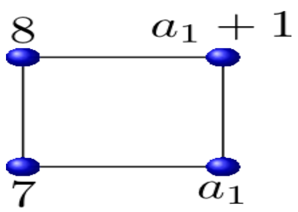

Fig: 1:6

$\prod_{a_k}^{a_k+1\ c_k+1}$

$$\gcd(a_k, c_k + 1) = \gcd(a_k, a_k - 1) = 1,$$
$$\gcd(c_k, a_k + 1) = \gcd(a_k - 2, a_k + 1) = \gcd(3, 6k - 2) = 1$$

,

$$\gcd(c_k, a_k + 1) = \gcd(6k - 5, 6_k + 4) = gcd\ (9, 6_k + 4) = 1,$$
$$\gcd(a_{k+1}, c_k + 1) = \gcd(6k_k + 3, 6_k - 4) = \gcd(7, k + 4 - 2) = 1 \text{ (as } k\ satisfies)$$

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which we call a_k -square and b_k -square, respectively. It turns out that in some situations these two do not fulfil the cross condition, in which cases we need to replace some of the labels already assigned. These two types of squares will be handled in the following subsections.

1.1 a_k -squares

1.1.1 a_k -squares with $7 \nmid k+3$ or $5 \nmid k+2$

We show that in this case, a_k -squares fulfil the cross condition. Set

$$d_1 := \gcd(a_k, a_{k+1}+1) = \gcd(6k-3, 6k+4) = \gcd(7, k+3)$$
$$d_2 := \gcd(a_{k+1}, a_k+1) = \gcd(6k+3, 6k-2) = \gcd(5, k+3)$$

We have $7 \nmid k+3$ or $5 \nmid k+2$. First suppose that $7 \nmid k+3$. So $d_1 = 1$. For a contradiction, assume that $d_2 > 1$, and so $5 \mid k+3$. If it happens that $6k+1 = c_{k+1} \in C$, then $\ell = k+1$ satisfies (功), so $5 \mid k+4$ or $5 \mid k+1$ which is impossible. In addition, $6k+1 \neq 7$. It follows that $6k+1 \in D$. If $7 \nmid k+1$, then $\ell = k$ satisfies ((D), so $6k+1$ must be already between a_k and a_{k+1}, a contradiction. If $7 \mid k+1$, then $\ell = k$ satisfies (𝕀), so c_k exists and must be already between a_k and a_{k+1}, again a contradiction. It follows that $d_2 = 1$.

Now assume that $7 \mid k+3$ and $5 \nmid k+2$. Again $6k+1 \in D$, since otherwise $6k+1 = c_{k+1} \in C$ which means that $\ell = k+1$ satisfies (𝟙), hence $7 \mid k+2$ or $7 \mid k+4$ which is impossible. We also have $5 \nmid k$, since otherwise c_k exists, so it must had been placed between a_k and a_{k+1}, a contradiction. It turns out that $\ell = k$ satisfies (2), so $6k+1$ must be already between a_k and a_{k+1}, again a contradiction.

1.1.2 a_k -squares with $7 \mid k+3$ and $5 \mid k+2$

In this case we have $k = 35q - 17$ for some positive integer q. Let $m = a_k = 210q - 105$. It turns out that in this case a_k -square do not fulfil the cross condition. To overcome this obstacle, we exchange some of the labels around a_k.

For $k = 35q - 17$, it can be essily seen that c_{k+1} does not exist and $6(k+1)+1 = 210q - 95$ belongs to D and $\ell = k+1$ satisfies (12). Hence $210q - 95$ is already inserted betwoen a_{k+1} and a_{k+2}

Case 1. $11 \nmid m$ and $11m > 2n$.

- If $210q - 94 \leq 2n$, then we change the original labeling to the following:

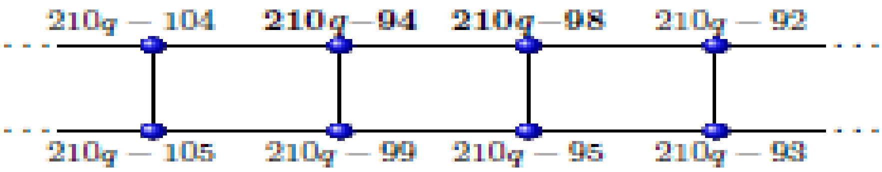

Fig: 1:7

This changes the labeling of three consecutive squares above and as a result all three new squares fulfil the cross condition.

- If $210q - 94 > 2n$, then we change the labeling as follows:

Case 2. $11 \nmid m$ and $11m < 2n$.

Let t be the maximum integer with $11^t m < 2n$.

If $11 \nmid m - 1$, then we change the original labeling to the following:

If $11 \mid m - 1$, then we change the original labeling to the following:

Case 3. 11 | m

As $11 \mid m$, it follows that there are integers s, q' such that $m = 11^s(210q' - 105)$ where $11 \nmid 210q' - 105$. So as we did in Case $2, m$ and $m - 1$ has been already replaced to elsewhere. Hence we can change the labels as follows:

1. 2 b_k -squares

2.2.1 b_k -squares with $7 \nmid k + 1$ or $5 \nmid k + 1$

We show that in this case b_k -squares fulfil the cross condition. Set

$$d_1 := \gcd(b_{k+1}, b_k + 1) = \gcd(5, k)$$
$$d_2 := \gcd(b_k, b_{k+1} + 1) = \gcd(7, k + 1)$$

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For a contradiction, suppose that $d_1 > 1$, so $5 \mid k$. We have $6k + 1 \in D$ since otherwise, as $6k + 1 \neq 7$, we have $6k + 1 = c_{k+1}$ and so $\ell = k + 1$ must satisfy (II) which is impossible. As $\ell = k$ does not satisfy ([2), $6k + 1$ must had been inserted between b_k and b_{k+1}, a contradiction. Therefore, $d_1 = 1$

If $7 \nmid k + 1$, then $d_2 = 1$. Assume that $7 \mid k + 1$ and $5 \nmid k + 1$. Since $\ell = k + 1$ does not satisfy

(II), we have necessarily $6k + 1 \in D$. As $5 \nmid k + 1, \ell = k$ does not satisfy (a), and so $6k + 1$ is already inserted between b_k and b_{k+1}, a contradiction.

2.2.2 b_k -squares with $7 \mid k + 1$ and $5 \mid k + 1$

In this ease we have $k = 35q - 1$ for some positive integer q. Let $m = b_{k+1} + 1 = 210$ q. It turns out that in this case b_k -squares do not fulfil the cross condition and as before we exchange some of the labels around b_k –

Case 1. $11 \nmid m$ and $11m \geq 2n$.

We change the original labeling to the following:

Case 2. $11 \nmid m$ and $11m < 2n$.

Let t be the maximum integer with $11^t m < 2n$.

If $11 \nmid m + 1$, then we change the original labeling to the following:

If $11 \mid m + 1$, then we change the original labeling

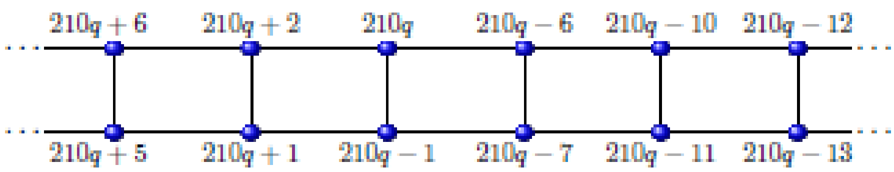

Fig:1:8

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to the following:

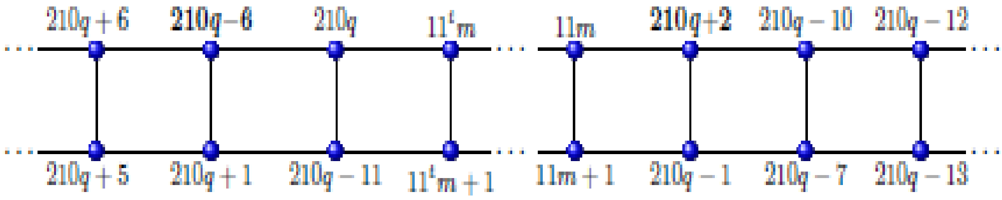

Fig:1;9

Case 3. 11 | m.

If $m < 2n$, then $m = 11^s(210q')$ for some positive integers s, q' with $11 \nmid 210q'$. So as we did in Case 2, m and $m+1$ has been already replaced to elsewhere. Hence we can change the labels as follows:

If $m = 2n$, then we insert 1 between b_k and b_{k+1} :

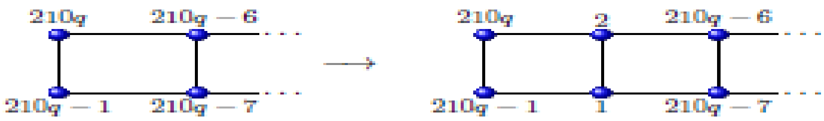

Fig: 1;10

Finally if the labels 1 and 2 have not been used, we may insert 1 between any two vertioes.

Conclusion:

Prime labeling is an assignment of integers are considerable numbers of open problems and to the vertices which are relatively prime. The theory literatures are available for prime labeling of various behind the graph labeling were introduced by Rosa types of graphs. Our previous researches are focused in the 1960s. A huge number of prime labeling on the prime labeling for Tripartite graphs, Roach research works have been discovered for various graph, Scorpion graph and Crab graph [2].[3],[7]. The types of graphs today after undergoing different present work is aimed to investigate some new results methods. In this work, the prime labeling method has on prime labeling for a special graph called Circular been introduced for the circular ladder graph CL_n

Ladder graph with n number of vertices in one cycle with $2n$ vertices when n is an even. The graph where n is even. obtained by using cartesian product of C_n with n vertices and path graph of the form P_2 is called We initiate with a simple finite connected undirected circular ladder graph and is denoted by CL_n (ie graph $G = (V(G), E(G))$ with $2n$ number of vertices. $C_n \times P_2 = CL_n$). We proved that CL_n is a prime.

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