

RADIO LEHMER-3 MEAN NUMBER OF SOME GRAPHS [1]

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Abstract:A radio Lehmer-3 mean labeling of a connected graph G is a one to one map h from the vertex set V (G) to the set of natural numbers N such that for two distinct vertices x and y of G, $d(x,y) + \left\lceil \frac{h(x)^3+h(y)^3}{h(x)^2+h(y)^2} \right\rceil \geq 1 + \text{diam}(G)$. The radio Lehmer-3 mean numer of h, $rl_{3mn}(h)$ is the maximum number assigned to any vertex of G. The radio lehmer-3 mean number of G, $rl_{3mn}(G)$ is the minimum value of $rl_{3mn}(h)$ taken over all radio lehmer-3 mean labeling h of G. In this paper we investigateradio Lehmer-3 mean labeling of some graphs.

Keywords: Radio lehmer-3 mean labeling, Diameter, comb etc.

Introduction

Throughout this paper we consider finite, simple, undirected and connected graphs. Let V(G) and E(G) respectively denote the vertex set and edge set of G. Radio labeling, or multilevel distance labeling, is motivated by the channel assignment problem for radio transmitters Ponraj et al. [6] introduced the notion of radio meanlabelingofgraphsandinvestigatedradiomeannumberofsomegraphs.For standard terminology and notations we follow Harary [3] and Gallian [1]. The distance between two vertices x and y of G is denoted by d(x, y) and diam(G) indicate the diameter ofG.In this paper we introduce the radio lehmer 3 mean labeling of some graphs.

Definition:

A radio Lehmer-3 mean labeling of a connected graph G is a one to one map h from the vertex set V (G) to the set of natural numbers N such that for two distinct vertices x and y of G, $d(x,y) + \left\lceil \frac{h(x)^3+h(y)^3}{h(x)^2+h(y)^2} \right\rceil \geq 1 + \text{diam}(G)$. The radio Lehmer-3 mean numer of h, $rl_{3mn}(h)$ is the maximum number assigned to any vertex of G. The radio lehmer-3 mean number of G, $rl_{3mn}(G)$ is the minimum value of $rl_{3mn}(h)$ taken over all radio lehmer-3 mean labeling h of G.

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Main Result

Theorem
$rl_{3mn}(P_m \odot k_{1,2})$= 4m-3, n≥ 2

Proof.

Let $V(P_m \odot k_{1,2}) = \{u_j, v_j, w_j\ 1 \le j \le m\}$ *and*

$$E\{P_m \odot k_{1,2})\} = \{u_j u_{j+1};\ 1 \le j \le m-1\} \cup \{u_j v_j;\ u_j w_j;\ 1 \le j \le m\} \cup \{u_m v_1\}$$

and diam($P_m \odot k_{1,2}$) = m + 1

Define a function $h : V(P_m \odot k_{1,2}) \rightarrow N$by

$$h(u_j) = 3m + j - 3 \qquad 1 \le j \le m$$

$$h(v_j) = m + j - 3 \qquad 1 \le j \le m$$

$$h(w_j) = 2m + j - 3 \qquad 1 \le j \le m$$

NextwechecktheradioLehmer-3meanconditionfor*h*.

Case(a).Takethepair(u_j, u_k),$1 \le j \le m-1, j+1 \le k \le m$

$$d(u_j, u_k) + \left\lceil \frac{h(u_j)^3 + h(u_k)^3}{h(u_j)^2 + h(u_k)^2} \right\rceil = 1 + \left\lceil \frac{(3m+j-3)^3 + (3m+k-3)^3}{(3m+j-3)^2 + (3m+k-3)^2} \right\rceil \ge m + 2 \ge 1 + \text{diam}(G)$$

Case(b).Takethepair(v_j, v_k),$1 \le j \le m-1, j+1 \le k \le m$

$$d(v_j, v_k) + \left\lceil \frac{h(v_j)^3 + h(v_k)^3}{h(v_j)^2 + h(v_k)^2} \right\rceil = 1 + \left\lceil \frac{(m+j-3)^3 + (m+k-3)^3}{(m+j-3)^2 + (m+k-3)^2} \right\rceil \ge m + 2$$

Case(c).Takethepair(w_j, w_k),$1 \le j \le m-1, j+1 \le k \le m$

$$d(w_j, w_k) + \left\lceil \frac{h(w_j)^3 + h(w_k)^3}{h(w_j)^2 + h(w_k)^2} \right\rceil = 1 + \left\lceil \frac{(2m+j-3)^3 + (2m+k-3)^3}{(2m+j-3)^2 + (2m+k-3)^2} \right\rceil \ge m + 2$$

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Case(d).Takethepair$(v_j, u_k), 1 \le j \le m, 1 \le k \le m$

$$d(v_j, u_k) + \left\lceil \frac{h(v_j)^3 + h(u_k)^3}{h(v_j)^2 + h(u_k)^2} \right\rceil = 1 + \left\lceil \frac{(m + j - 3)^3 + (3m + k - 3)^3}{(m + j - 3)^2 + (3m + k - 3)^2} \right\rceil \ge m + 2$$

Case(e).Takethepair$(w_j, u_k), 1 \le j \le m, 1 \le k \le m$

$$d(w_j, u_k) + \left\lceil \frac{h(w_j)^3 + h(u_k)^3}{h(w_j)^2 + h(u_k)^2} \right\rceil = 1 + \left\lceil \frac{(2m + j - 3)^3 + (3m + k - 3)^3}{(2m + j - 3)^2 + (3m + k - 3)^2} \right\rceil \ge m + 2$$

Case(f).Takethepair(v_j, w_k),$1 \le j \le m, 1 \le k \le m$

$$d(v_j, w_k) + \left\lceil \frac{h(v_j)^3 + h(w_k)^3}{h(v_j)^2 + h(w_k)^2} \right\rceil = 2 + \left\lceil \frac{(m + j - 3)^3 + (2m + k - 3)^3}{(m + j - 3)^2 + (2m + k - 3)^2} \right\rceil \ge m + 2$$

Thus all the pair of vertices satisfies the radio Lehmer-3 mean condition.
Hence $rl_{3mn}(P_m \odot k_{1,2})$= 4m-3, n≥ 2

Theorem
$rl_{3mn}(W_m \odot k_{1,2})$.= 3m+1,m≥ 3

Proof.

Let $V\ (W_m \odot k_{1,2}) = \{s, u_j, v_j, w_j\ 1 \le j \le m\}\ and$

$$E\{P_m \odot k_{1,2})\} = \{u_j u_{j+1};\ 1 \le j \le m-1\} \cup \{u_j v_j;\ u_j w_j, su_j;\ 1 \le j \le m\}$$

and diam$(W_m \odot k_{1,2}) = 4$

Define a function $h : V(W_m \odot k_{1,2}) \to N$by

$$h(u_j) = 2m + j \qquad 1 \le j \le m$$

$$h(v_j) = j \qquad 1 \le j \le m$$

$$h(w_j) = m + j \qquad 1 \le j \le m$$

$$h(s) = 3m + 1$$

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NextwecheckttheradioLehmer-3meanconditionfor$h$.

Case(a).Takethepair(u_j ,u_k),$1 \leq j \leq m-1, j+1 \leq k \leq m$

$$d(u_j, u_k) + \left\lceil \frac{h(u_j)^3 + h(u_k)^3}{h(u_j)^2 + h(u_k)^2} \right\rceil = 1 + \left\lceil \frac{(2m+j)^3 + (2m+k)^3}{(2m+j)^2 + (2m+k)^2} \right\rceil \geq 5 \geq 1 + \text{diam}(G)$$

Case(b).Takethepair(v_j ,v_k),$1 \leq j \leq m-1, j+1 \leq k \leq m$

$$d(v_j, v_k) + \left\lceil \frac{h(v_j)^3 + h(v_k)^3}{h(v_j)^2 + h(v_k)^2} \right\rceil = 1 + \left\lceil \frac{(j)^3 + (k)^3}{(j)^2 + (k)^2} \right\rceil \geq 5$$

Case(c).Takethepair(w_j ,w_k),$1 \leq j \leq m-1, j+1 \leq k \leq m$

$$d(w_j, w_k) + \left\lceil \frac{h(w_j)^3 + h(w_k)^3}{h(w_j)^2 + h(w_k)^2} \right\rceil = 1 + \left\lceil \frac{(m+j)^3 + (m+k)^3}{(m+j)^2 + (m+k)^2} \right\rceil \geq 5$$

Case(d).Takethepair(v_j ,u_k), $1 \leq j \leq m, 1 \leq k \leq m$

$$d(v_j, u_k) + \left\lceil \frac{h(v_j)^3 + h(u_k)^3}{h(v_j)^2 + h(u_k)^2} \right\rceil = 1 + \left\lceil \frac{(j)^3 + (2m+k)^3}{(j)^2 + (2m+k)^2} \right\rceil \geq 5$$

Case(e).Takethepair(w_j ,u_k), $1 \leq j \leq m, 1 \leq k \leq m$

$$d(w_j, u_k) + \left\lceil \frac{h(w_j)^3 + h(u_k)^3}{h(w_j)^2 + h(u_k)^2} \right\rceil = 1 + \left\lceil \frac{(m+j)^3 + (2m+k)^3}{(m+j)^2 + (2m+k)^2} \right\rceil \geq 5$$

Case(f).Takethepair(v_j ,w_k),$1 \leq j \leq m, 1 \leq k \leq m$

$$d(v_j, w_k) + \left\lceil \frac{h(v_j)^3 + h(w_k)^3}{h(v_j)^2 + h(w_k)^2} \right\rceil = 2 + \left\lceil \frac{(j)^3 + (m+k)^3}{(j)^2 + (m+k)^2} \right\rceil \geq 5$$

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Case(g).Takethepair(s,u_j), $1 \leq j \leq m$

$$d(s,u_k) + \left\lceil \frac{h(s)^3+h(u_j)^3}{h(s)^2+h(u_j)^2} \right\rceil = 1 + \left\lceil \frac{(3m+1)^3+(2m+j)^3}{(3m+1)^2+(2m+j)^2} \right\rceil \geq 5$$

Case(h).Takethepair(s,v_j),$1 \leq j \leq m$

$$d(s,v_k) + \left\lceil \frac{h(s)^3+h(v_j)^3}{h(s)^2+h(v_j)^2} \right\rceil = 1 + \left\lceil \frac{(3m+1)^3+(j)^3}{(3m+1)^2+(j)^2} \right\rceil \geq 5$$

Case(i).Takethepair(s,w_j), $1 \leq j \leq m$

$$d(s,w_k) + \left\lceil \frac{h(s)^3+h(w_j)^3}{h(s)^2+h(w_j)^2} \right\rceil = 1 + \left\lceil \frac{(3m+1)^3+(m+j)^3}{(3m+1)^2+(m+j)^2} \right\rceil \geq 5$$

Thus all the pair of vertices satisfies the radio Lehmer-3 mean condition.

Hence $rl_{3mn}(W_m \odot k_{1,2})$.= 3m+1,

Theorem

$rl_{3mn}(K_m \odot k_{1,2})$= 3m, n≥ 2

Proof.

Let $V(K_m \odot k_{1,2}) = \{u_j, v_j, w_j \; 1 \leq j \leq m\}$ *and*

$$E\{P_m \odot k_{1,2})\} = \{u_j u_{j+1};\ 1 \leq j \leq m-1\} \cup \{u_j v_j;\ u_j w_j;\ 1 \leq j \leq m\}\{u_1 u_m\}$$

and diam$(K_m \odot k_{1,2}) = 3$

Define a function $h : V(K_m \odot k_{1,2}) \to N$by

$$h(u_j) = 2m + j \qquad 1 \leq j \leq m$$

$$h(v_j) = 2j - 1 \qquad 1 \leq j \leq m$$

$$h(w_j) = 2j \qquad 1 \leq j \leq m$$

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NextwechecktheradioLehmer-3meanconditionfor$h$.

Case(a).Takethepair(u_j, u_k),$1 \le j \le m-1, j+1 \le k \le m$

$$d(u_j, u_k) + \left\lceil \frac{h(u_j)^3 + h(u_k)^3}{h(u_j)^2 + h(u_k)^2} \right\rceil = 1 + \left\lceil \frac{(2m+j)^3 + (2m+k)^3}{(2m+j)^2 + (2m+k)^2} \right\rceil \ge 4 = 1 + \text{diam}(G)$$

Case(b).Takethepair(v_j, v_k),$1 \le j \le m-1, j+1 \le k \le m$

$$d(v_j, v_k) + \left\lceil \frac{h(v_j)^3 + h(v_k)^3}{h(v_j)^2 + h(v_k)^2} \right\rceil = 3 + \left\lceil \frac{(2j-1)^3 + (2k-1)^3}{(2j-1)^2 + (2k-1)^2} \right\rceil \ge 4$$

Case(c).Takethepair(w_j, w_k),$1 \le j \le m-1, j+1 \le k \le m$

$$d(w_j, w_k) + \left\lceil \frac{h(w_j)^3 + h(w_k)^3}{h(w_j)^2 + h(w_k)^2} \right\rceil = 3 + \left\lceil \frac{(2j)^3 + (2k)^3}{(2j)^2 + (2k)^2} \right\rceil \ge 4$$

Case(d).Takethepair(v_j, u_k), $1 \le j \le m, 1 \le k \le m$

$$d(v_j, u_k) + \left\lceil \frac{h(v_j)^3 + h(u_k)^3}{h(v_j)^2 + h(u_k)^2} \right\rceil = 1 + \left\lceil \frac{(2j-1)^3 + (2m+k)^3}{(2j-1)^2 + (2m+k)^2} \right\rceil \ge 4$$

Case(e).Takethepair(w_j, u_k), $1 \le j \le m, 1 \le k \le m$

$$d(w_j, u_k) + \left\lceil \frac{h(w_j)^3 + h(u_k)^3}{h(w_j)^2 + h(u_k)^2} \right\rceil = 1 + \left\lceil \frac{(2j)^3 + (2m+k)^3}{(2j)^2 + (2m+k)^2} \right\rceil \ge 4$$

Case(f).Takethepair(v_j, w_k),$1 \le j \le m, 1 \le k \le m$

$$d(v_j, w_k) + \left\lceil \frac{h(v_j)^3 + h(w_k)^3}{h(v_j)^2 + h(w_k)^2} \right\rceil = 2 + \left\lceil \frac{(2j-1)^3 + (2k)^3}{(2j-1)^2 + (2k)^2} \right\rceil \ge 4$$

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Thus all the pair of vertices satisfies the radio Lehmer-3 mean condition.

Hence $rl_{3mn}\,(K_m \odot k_{1,2})$= 3m, n≥ 2

Conclusion:

In this paper we discussed about radio lehmer-3 mean labeling for some standard graphs,More rsults will be done in the further research article.

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