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AN ADAPTIVE GENERAL COMBINED RESTRICTED RIDGE REGRESSION ESTIMATOR FOR LINEAR REGRESSION MODEL UNDER MULTICOLLINEARITY

Dr.Bandi Ramanjineyulu
Manager, Clinical Statistics, GlaxoSmithKline, Bangalore, Karnataka, India
Email id : ramanji.bandi@gmail.com

ABSTRACT

The introduction by Hoerl and Kennard (1970) of a ridge regression estimator to deal with the problem of multicollinearity in regression analysis has been followed by a number of research articles in the statistics literature. The defined a class of estimators characterized by a scalar called Ridge parameter. By reducing the linear regression model to its canonical form, they proposed the general Ridge regression estimator.

In the present study, an adaptive general combined restricted Ridge regression estimator based on an iterative procedure has been proposed by combining the restricted least squares estimator with a ridge regression estimator.

I. INTRODUCTION:

A serious problem that can occur in regression analysis is the presence of muticollinearity among the independent variables in a regression equation. The

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multicollinear problem may arise when some or all of the explanatory variables in a regression are highly correlated with one another.

The main consequences of multicollinearity are:

i) The precision of estimation falls so that it becomes very difficult to obtain precise estimates of the regression coefficients. The loss of precision has three aspects.

   a) Specific estimates have very large errors.
   b) These errors may be highly correlated with one another.
   c) The sampling variances of the coefficients will be very large.

ii) Estimates of regression coefficients become very sensitive to particular sets of sample data and the addition of a few more observations or deletion of a few observations can sometimes produce dramatical shifts in some of the coefficients.

iii) If multicollinearity is high, one may obtain a high value of R^2 and very few estimated regression coefficients are statistically significant.

In presence of multicollinearity, certain biased estimation procedures like Ridge regression, Generalized inverse estimator, principal component regression, Liu estimator are used to improve the efficiency of ordinary least squares (OLS) estimates in the linear regression model.

The introduction by Hoerl and Kennard (1970) of a ridge regression estimator to deal with the problem of multicollinearity in regression analysis has been followed by a number of research papers in the Statistical literature. They defined a class of estimator's characterized by a scalar called 'Ridge parameter'. First, they presented a scheme for choosing the scalar with the help of 'Ridge Trace' and observed that the

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regression coefficient estimators stabilize for small values of the Ridge Parameter. Then, by reducing the linear regression model to its canonical form, they defined the general ridge regression estimator.

The main contributions made to the field of inference in linear regression models under multicollinearity by Farrar and Glauber (1967), Rama Sastry (1970), Marquardt and Snee (1975), Webster, Gunst and Mason (1975), Krishna Kumar (1975), Vinod (1979), Willan and Watts (1978) Swamy, Mehta, Thurman and Iyengar (1985), Judge and Griffiths (1985), Mehta, Swamy and Iyengar (1993) and others.

The present study some new ridge regression inferential techniques have been developed to estimate the parameters of linear regression models under the problem of multicollinearity.

II. RESTRICTED LEAST SQUARES ESTIMATOR:

Consider a standard linear regression model

$$Y_{n\times1} = X_{n\times k}\beta_{k\times1} + \in_{n\times1} \qquad (2.1)$$

$$E[\in] = 0, E\left[\in\in^{1}\right] = \sigma^{2}I_{n}$$

Assuming that two or more explanatory variables in X are closely related so that the model is subject to the problem of multicollinearity.

A set of a prior q restrictions on the parameters may be written as

$$R\beta = r \qquad (2.2)$$

Where R is a $(q \times K)$ matrix of known prior information design matrix that expresses the structure of information on the individual parameters or some linear combinations of the parameters.

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r is a $(q \times 1)$ vector of known elements

β is a $(K \times 1)$ vector of unknown parameters

The restricted least square estimator $(\hat{\beta}_R)$ for β can be obtained by minimizing $(Y - X\beta)^1(Y - X\beta)$ subject to the restrictions $R\beta = r$. It is given by

$$\hat{\beta}_R = \hat{\beta} + (X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}(r - R\hat{\beta}) \tag{2.3}$$

where $\hat{\beta} = (X^1X)^{-1}X^1Y$ is the unrestricted OLS estimator of β.

$$E\left(\hat{\beta}_R\right) \neq \beta, \text{ unless } R\beta = r \text{ holds}$$

The dispersion matrix of $\hat{\beta}_R$ is given by

$$\text{Var}\left(\hat{\beta}_R\right) = \sigma^2(X^1X)^{-1} - \sigma^2(X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}R(X^1X)^{-1} \tag{2.4}$$

Since, $\text{Var}(\hat{\beta}) = \sigma^2(X^1X)^{-1}$, one may be concluded that the $(\hat{\beta}_R)$ has a smaller sampling variance than the estimator $\hat{\beta}$.

Also, the MSE of $\hat{\beta}_R$ is given by

$$\text{MSE}\left[\hat{\beta}_R\right] = \sigma^2 \ \text{tr}\left[(X^1X)^{-1} - (X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}R(X^1X)^{-1}\right]$$

$$+(r - R\beta)^1\left[R(X^1X)^{-1}R^1\right]^{-1}R(X^1X)^{-2}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}(r - R\beta) \tag{2.5}$$

III. ORDINARY RIDGE REGRESSION ESTIMATOR:

Consider the family of ordinary ridge regression estimators.

$$\hat{\beta}(\delta) = \left[X^1X + \delta I_k\right]^{-1}X^1Y, \delta \geq 0 \tag{3.1}$$

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$$\text{or } \hat{\beta}(\delta) = \left[I_k + \delta(X^1X)^{-1}\right]^{-1}\hat{\beta} \qquad (3.2)$$

$E\left[\hat{\beta}(\delta)\right] \neq \beta$ and the Bias of $\hat{\beta}(\delta)$ is given by

$$\text{Bias}\left[\hat{\beta}(\delta)\right] = \delta\left[(X^1X) + \delta I_k\right]^{-1}\beta \qquad (3.3)$$

$$\text{Also, } \text{Var}\left[\hat{\beta}(\delta)\right] = \sigma^2\left[X^1X + \delta I_k\right]^{-1}(X^1X)\left[X^1X + \delta I_k\right]^{-1} \qquad (3.4)$$

Thus, $\left[\text{Var}(\hat{\beta}) - \text{Var}(\hat{\beta}(\delta))\right]$ is non negative definite matrix for $\delta \geq 0$

It should be noted that the $\hat{\beta}(\delta)$ can be obtained by minimising $\beta^1\beta$ subject to $(\beta - \hat{\beta})^1 X^1X(\beta - \hat{\beta}) = \phi$

Where ϕ is fixed value

OPTIMUM VALUE OF RIDGE PARAMETER:

Let G is an ortho normal matrix whose columns are normalized Eigen

vectors of X^1X

i.e., $G^1 = I$ and

$$G^1X^1XG = \begin{bmatrix} \lambda_1 & 0\ldots.. & 0 \\ 0 & \lambda_2\ldots. & 0 \\ \vdots & \vdots & \\ 0 & 0 & \lambda_k \end{bmatrix} \text{and } \lambda_1 \geq \lambda_2 \geq \ldots. \geq \lambda_k \qquad (3.5)$$

$$\text{Write } Y = X\beta + \in$$

$$= XGG^1\beta + \in$$

$$\Rightarrow Y = Z\Gamma + \in \qquad (3.6)$$

Where $Z = XG$, $\Gamma = G^1\beta$

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Now, the OLS estimator of Γ is given by

$$\hat{\Gamma} = \left[G^1X^1XG\right]^{-1} G^1X^1Y$$

$$= \left[G^1X^1XG\right]^{-1} G^1X^1X\hat{\beta}$$

$$= \left[G^1X^1XG\right]^{-1} G^1X^1XG\,G^1\hat{\beta}$$

$$\Rightarrow \hat{\Gamma} = G^1\hat{\beta} \qquad (3.7)$$

An ordinary ridge regression estimator of Γ is given by

$$\hat{\Gamma}(\delta) = G^1\hat{\beta}(\delta) \qquad (3.8)$$

Also, $\hat{\Gamma}(\delta) = \left[I + \delta(Z^1Z)^{-1}\right]^{-1}\hat{\Gamma}$

$$\hat{\Gamma}(\delta) = \left[I + \delta(G^1X^1XG)^{-1}\right]^{-1}\hat{\Gamma} \qquad (3.9)$$

Since, $\hat{\Gamma} = G^1\hat{\beta}$, we have

$$\hat{\Gamma}(\delta) = \left[I + \delta(G^1X^1XG)^{-1}\right]^{-1} G^1\left[I + \delta(X^1X)^{-1}\right]\hat{\beta}(\delta) \qquad (3.10)$$

It can be easily seen that (3.7) and (3.9) are equivalent.

Since $(\varsigma^1X^1X\varsigma)$ is diagonal matrix, it follows that

$$\hat{\Gamma}_j(\delta) = \left[\frac{\lambda_j}{\lambda_j + \delta}\right]\hat{\Gamma}_j \quad j = 1,2,\ldots,k \qquad (3.11)$$

An optimum value of δ is given by

$$\delta_{opt} = K\frac{\hat{\sigma}^2}{\hat{\beta}^1\hat{\beta}}, \, , \qquad (3.12)$$

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where $\hat{\sigma}^2 = \dfrac{(Y - X\hat{\beta})^1(Y - X\hat{\beta})}{n - K}$

When $(X^1X) = I$, the value of δ that minimizes the sum of the mean square errors is equal to $\dfrac{K\sigma^2}{\beta^1\beta}$ (3.13)

IV. PROPOSED COMBINED RESTRICTED RIDGE REGRESSION ESTIMATOR:

Replacing $\hat{\beta}$ by $\hat{\beta}_R$, the restricted ridge regression estimator for β is given by

$$\hat{\beta}_R(\delta) = \left[I_k + \delta(X^1X)^{-1}\right]^{-1}\hat{\beta}_R, \delta \geq 0 \quad (4.1)$$

Where $\hat{\beta}_R$ is the Restricted least squares estimator of β.

$$\text{If } \delta = 0, \text{then } \hat{\beta}_R(\delta) = \hat{\beta}_R 0 \quad (4.2)$$

The ordinary ridge regression estimator $\hat{\beta}(\delta)$ can be obtained by minimizing $\beta^1\beta$ subject to $(\beta - \hat{\beta}_R)^1X^1X(\beta - \hat{\beta}_R) = \phi_R$, where ϕ_R is fixed.

Using the linear restrictions Rβ=r, one can obtain

$$E\left[\hat{\beta}_R(\delta)\right] = \left[I_k + \delta(X^1X)^{-1}\right]^{-1}\beta + \left[I_k + \delta(X^1X)^{-1}\right]^{-1}(X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}(r - R\beta) \quad (4.3)$$

The restricted ridge regression estimator $\hat{\beta}_R(\delta)$ is always a biased estimator of β unless δ=0 and $(r - R\beta) = 0$, a null vector.

Further,

$$Var\left[\hat{\beta}_R(\delta)\right] = \sigma^2\left[I_k + \delta(X^1X)^{-1}\right]^{-1}\left[(X^1X)^{-1} - (X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}R(X^1X)^{-1}\right]\left[I_k + \delta(X^1X)^{-1}\right]^{1^{-1}}$$

(4.4)

Also, the MSE of $\hat{\beta}_R(\delta)$ is given by

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$$MSE\left[\hat{\beta}_R(\delta)\right] = \sigma^2 tr\left[TL\,T^1\right] + \left[T(X^1X)^{-1}\Delta^* - \delta\left[X^1X + \delta I_k\right]^{-1}\beta\right]^1$$
$$\left[T(X^1X)^{-1}\Delta^k - \delta\left[X^1X + \delta I_k\right]^{-1}\beta\right] \quad (4.5)$$

Where $T = \left[I_k + \delta(X^1X)^{-1}\right]^{-1}, L = (X^1X)^{-1} - (X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}R(X^1X)^{-1}$

$$\Delta^* = R^1\left[R(X^1X)^{-1}R^1\right]^{-1}(r - R\beta)$$

under the restrictions are true, i.e., $(r - R\beta) = 0$, the

$MSE\left[\hat{\beta}_R(\delta)\right]$ reduces to

$$MSE\left[\hat{\beta}_R(\delta)\right] = \sigma^2 tr\left[TLT^1\right] + \delta^2\beta^1\left[X^1X + \delta I_k\right]^{-2}\beta \quad (4.6)$$

Consider an optimum value of δ based on Restricted least squares estimator is given by

$$\hat{\delta}_R = K\frac{\hat{\sigma}_R^2}{\hat{\beta}_R^1\hat{\beta}_R} \quad (4.7)$$

Where $$\hat{\sigma}_R^2 = \frac{(Y - X\hat{\beta}_R)^1(Y - X\hat{\beta}_R)}{n - K} \quad (4.8)$$

$$\hat{\beta}_R = \hat{\beta} + (X^1X)^1R^1\left[R(X^1X)^{-1}R^1\right]^{-1}(r - R\hat{\beta}) \quad (4.9)$$

Here, $\hat{\beta}$ is the OLS estimator of β.

Now the proposed combined restricted ridge regression estimator for β is given by

$$\hat{\beta}_R(\hat{\delta}_R) = \left[I_k + \hat{\delta}_R(X^1X)^{-1}\right]^{-1}\hat{\beta}_R, \hat{\delta}_R \geq 0 \quad (4.10)$$

We have $\hat{\beta}_R(\hat{\delta}_R) = \hat{\beta}_R$ when $(\hat{\delta}_R) = 0$.

By considering the linear restrictions on parameters Rβ=r, one may obtain

$$E\left[\hat{\beta}_R(\hat{\delta}_R)\right] = \left[I_k + \hat{\delta}_R(X^1X)^{-1}\right]^{-1}\beta + \left[I_k + \hat{\delta}_R(X^1X)^{-1}\right]^{-1}(X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}(r - R\beta) \tag{4.11}$$

Thus , $\hat{\beta}_R(\hat{\delta}_R)$ is always a biased estimator of β unless

$\hat{\delta}_R = 0$ and $(r - R\beta) = 0$, a null vector

The variance covariance matrix of $\hat{\beta}_R(\hat{\delta}_R)$ is given by

$$Var\left[\hat{\beta}_R(\hat{\delta}_R)\right] = \sigma^2\left[I_k + \hat{\delta}_R(X^1X)^{-1}\right]^{-1}\left[(X^1X)^{-1} - (X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}R(X^1X)^{-1}\right]\left[I_k + \hat{\delta}_R(X^1X)^{-1}\right]^{-1^1} \tag{4.12}$$

The mean squared error of the proposed combined restricted ridge regression estimator is given by

$$MSE\left[\hat{\beta}_R(\hat{\delta}_R)\right] = \sigma^2 tr\left[T_R L T_R^1\right] + \left[T_R(X^1X)^{-1}\Delta^* - \hat{\delta}_R\left[X^1X + \hat{\delta}_R I_k\right]^{-1}\beta\right]^1\left\{T_R(X^1X)^{-1}\Delta^* - \hat{\delta}_R\left[X^1X + \hat{\delta}_R I_K\right]^{-1}\beta\right\} \tag{4.13}$$

Where $T_R = \left[I_K + \hat{\delta}_R(X^1X)^{-1}\right]^{-1}, L = (X^1X)^{-1} - (X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}R(X^1X)^{-1}$

$\Delta^* = R^1\left[R(X^1X)^{-1}R^1\right]^{-1}(r - R\beta)$

If the linear restrictions are true i.e., $(r - R\beta)$ =0 then the

MSE $\left[\hat{\beta}_R(\hat{\delta}_R)\right]$ reduces to

$$MSE\left[\hat{\beta}_R(\hat{\delta}_R)\right] = \sigma^2 tr\left[T_R L T_R^1\right] + \delta_R^2\left[(X^1X) + \hat{\delta}_R I_k\right]^{-2}\beta \tag{4.14}$$

To compare the performance of the proposed combined restricted ridge regression estimator $\hat{\beta}_R(\hat{\delta}_R)$ with (i) the restricted least squares estimator $\hat{\beta}_R$ and (ii) the ordinary Ridge Regression estimator $\hat{\beta}(\delta)$. We use the following two criteria based on Minimum sampling variance under the $(r - R\beta) = 0$

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(i) $Var\left[\hat{\beta}_R\right] - Var\left[\hat{\beta}_R(\hat{\delta}_R)\right] = \sigma^2\left[L - T_R L T_R^1\right]$

$$= \sigma^2 T_R\left[\hat{\delta}_R (X^1X)^{-1}L + \hat{\delta}_R L(X^1X)^{-1} + \hat{\delta}_R^2 (X^1X)^{-1}L(X^1X)^{-1}\right]T_R^1 \quad (4.15)$$

Since $(X^1X)^{-1}$ is positive definite and L is positive semi definite, it can easily be shown that all the characteristic roots of $(X^1X)^{-1}L$ are non negative so that $(X^1X)^{-1}L$ is a non negative definite matrix.

Since $(X^1X)^{-1}L(X^1X)^{-1}$ is positive definite and from (5.6.33) it can be seen that $\left[Var(\hat{\beta}_R) - Var\hat{\beta}_R(\hat{\delta}_R)\right]$ is positive semi definite for $\hat{\delta}_k \geq 0$.

Thus the sampling variance of the proposed combined restricted ridge regression estimator $\hat{\beta}_R(\hat{\delta}_R)$ is always less than or equal to that of the restricted least squares estimator $\hat{\beta}_R$.

(ii) Var $[\hat{\beta}(\hat{\delta}_R)] - Var[\hat{\beta}_R(\hat{\delta}_R)] = \left[X^1X + \hat{\delta}_R I_k\right]^{-1}(X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]R(X^1X)^{-1}\left[X^1X + \hat{\delta}_R I_k\right]^{-1}$

(4.16)

Which is a positive semi definite matrix for $\hat{\delta}_R \geq 0$.

Thus the proposed combined restricted ridge regression estimator $\hat{\beta}_R(\hat{\delta}_R)$ is always superior to the ordinary ridge regression estimator $\hat{\beta}(\hat{\delta}_R)$ by the criterion of sampling variance.

Remarks: One can compare the performance of the proposed combined restricted ridge regression estimator with the restricted least squares estimator and an ordinary ridge regression estimator of β by means of the criteria of minimum mean squared error under the two cases of (i) The liner

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restrictions about the parameters are true i.e., $(r - R\beta) = 0$ and (ii) the linear restrictions about the parameters are not true i.e., $(r - R\beta) \neq 0$.

V. A NEW ADAPTIVE GENERAL COMBINED RESTRICTED RIDGE REGRESSION ESTIMATOR:

Consider a classical linear regression model

$$Y_{n\times 1} = X_{n\times k}\beta_{k\times 1} + \in_{n\times 1} \quad (5.1)$$

$$E[\in] = 0 \text{ and } E\left[\in\in^1\right] = \sigma^2 I_n$$

$\in$ follows multivariate normal distribution N[0, $\sigma^2 I_n$].

The OLS estimator of β is given by $\hat{\beta} = (X^1X)^{-1}X^1Y$

Suppose that the set of a priori linear restrictions on the parameters are represented by.

$$R_{q\times K}\beta_{K\times 1} = r_{q\times 1} \quad (5.2)$$

The restricted least squares estimator of β is given by

$$\hat{\beta}_R = \hat{\beta} + (X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}(r - R\hat{\beta}) \quad (5.3)$$

It should be noted that the $\hat{\beta}_R$ can be obtained by minimizing $(Y - X\beta)^1(Y - X\beta)$ subject to the restrictions given in (5.7.2),

We have, $E\left[\hat{\beta}_R\right] \neq \beta$ unless (5.7.2) holds. Also, the variance covariance matrix of $\hat{\beta}_R$ is given by.

$$Var(\hat{\beta}_R) = \sigma^2\left[(X^1X)^{-1} - (X^1X)^{-1}R^1(R(X^1X)^{-1}R^1)^{-1} R(X^1X)^{-1}\right] \quad (5.4)$$

An ordinary ridge regression estimator for β is given by

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$$\hat{\beta}(\delta) = \left[X^1X + \delta I\right]^{-1} X^1Y, \delta \geq 0 \qquad (5.5)$$

By reducing the linear regression model (5.7.1) to its canonical form, the General Ridge Regression estimator for β may be defined.

Denote $(X^1X) = P$, where P=Diag$\{\lambda_1, \lambda_2, ..., \lambda_k\}$ is a $(K \times K)$ diagonal matrix.

The General Ridge Regression estimator for β is given by

$$\hat{\beta}(\Delta) = [P + \Delta]^{-1} X^1Y \qquad (5.6)$$

Where Δ is a diagonal matrix with non- negative elements $\delta_1, \delta_2, ..., \delta_k$ as characterizing scalars.

We write the OLS estimator of β using P as

$$\hat{\beta} = P^{-1}X^1Y \qquad (5.7)$$

From (5.7.6) and (5.77), the General Ridge Regression estimator for β can be written as

$$\hat{\beta}(\Delta) = [P + \Delta]^{-1} P\hat{\beta} = Q\hat{\beta} \qquad (5.8)$$

Where $Q = [P + \Delta]^{-1} P$

Replacing the OLS estimator $\hat{\beta}$ by the restricted least squares estimator $\hat{\beta}_R$ in (5.7.8) gives the restricted general ridge regression estimator for β as

$$\hat{\beta}_R(\Delta) = Q\hat{\beta}_R \qquad (5.9)$$

Where $Q = [P + \Delta]^{-1} P = \left[(X^1X) + \Delta\right]^{-1} (X^1X)$

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We have, $E\left[\hat{\beta}_R(\Delta)\right] = QE\left[\hat{\beta}_R\right] = Q\left[\beta + (X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}(r - R\beta)\right]$

$$= Q\beta + Q(X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}(r - R\beta) \qquad (5.10)$$

Thus $\hat{\beta}_R(\Delta)$ is always a biased estimator of β unless $\delta_i = 0, \forall i = 1, 2, \ldots\ldots, K$ and $(r - R\beta) = 0$, a null vector.

Further , $Var\left[\hat{\beta}_R(\Delta)\right] = E\left[\hat{\beta}_R(\Delta) - E[\hat{\beta}_R(\Delta)]\right]\left[\hat{\beta}_R(\Delta) - E[\hat{\beta}_R(\Delta)]\right]^1$

$$\Rightarrow Var\left[\hat{\beta}_R(\Delta)\right] = \sigma^2 \; Q\left[(X^1X)^{-1} - (X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}R(X^1X)^{-1}\right] Q^1 \qquad (5.11)$$

Also the Mean squared error of $\hat{\beta}_R(\Delta)$ is given by

$$MSE\left[\hat{\beta}_R(\Delta)\right] = \sigma^2 tr\left[QLQ^1\right] + \left\{Q(X^1X)^{-1}\eta^* - \Delta\left[X^1X + \Delta\right]^{-1}\beta\right\}^1 \left\{Q(X^1X)^{-1}\eta^* - \Delta\left[(X^1X) + \Delta\right]^{-1}\beta\right\}$$

$$(5.12)$$

Where $\eta^* = R^1\left[R(X^1X)^{-1}R^1\right]^{-1}(r - R\beta)$

$L = (X^1X)^{-1} - (X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}R(X^1X)^{-1}$

If $(r - R\beta) = 0$ then the $MSE\left[\hat{\beta}_R(\Delta)\right]$ reduces to

$$MSE\left[\hat{\beta}_R(\Delta)\right] = \sigma^2 tr\left[QLQ^1\right] + \Delta^2\beta^1\left[(X^1X) + \Delta\right]^{-2}\beta \qquad (5.13)$$

Consider an optimum value for δ_i as

$$\hat{\delta}_i = \frac{\sigma^2}{\beta_i^2}, i = 1, 2, \ldots\ldots K \qquad \text{[From (5.12]}$$

Using the restricted least squares estimator $\hat{\beta}_R$ and the restricted least squares residual sum of squares, the proposed optimum value for δ_i as

$$\hat{\delta}_{Ri} = \frac{S_R^2}{\hat{\beta}_{Ri}} = \frac{\sum_{j=1}^{n} e_{Rj}^2 / n-k-1}{\hat{\beta}_{Ri}^2}, i = 1, 2, ..., K \quad (5.14)$$

Where, $\hat{\beta}_{Ri}$ is the i^{th} element of the Restricted least squares estimator $\hat{\beta}_{Ri}$

$\sum_j e_{Rj}^2$ is the restricted least squares residual sum of squares.

We propose an Iterative procedure, under which we first compute $\hat{\delta}_{Ri}(0)$ as

$$\hat{\delta}_{Ri}(0) = \frac{S_R^2}{\hat{\beta}_{Ri}^2}, i = 1, 2, ..., K \quad (5.15)$$

and $$\hat{\beta}_R\left[\hat{\Delta}_0\right] = \left[P + \hat{\Delta}_0\right]^{-1} P\hat{\beta}_R \quad (5.16)$$

Where $\hat{\Delta}_R = \text{Diag}\ \{\hat{\delta}_{Ri}(0), \hat{\delta}_{R2}(0), ..., \hat{\delta}_{RK}(0)\}$

and P = (X^1X)

At the second stage, compute,

$$\hat{\delta}_{Ri}(1) \text{ as } \hat{\delta}_{Ri}(1) = \frac{S_R^2}{\hat{\beta}_{Ri}^2(\hat{\Delta}_0)}, i = 1, 2, ..., K \quad (5.17)$$

and $\hat{\beta}_R\ [\ \hat{\Delta}_1] = [\ P + \hat{\Delta}_1]^{-1}\ P\ \hat{\beta}_R$ (5.18)

Where $\hat{\Delta}_1 = \text{Diag}\ \{\hat{\delta}_{R1}(1), \hat{\delta}_{R2}(2), ..., \hat{\delta}_{RK}(1)\}$

We continue this Iterative process until we get the same values of $\hat{\delta}_{Ri}$'s with desired number of decimal points in two consecutive stages of the process.

Using the Iterative solutions of δ_{Ri}'s, the Ridge parameter matrix may be written as

$$\hat{\Delta}^* = \text{Diag}\left\{\hat{\delta}^*_{R1}, \hat{\delta}^*_{R2}, ..., \hat{\delta}^*_{RK}\right\} \quad (5.19)$$

Now the proposed adaptive general combined restricted ridge regression estimator for β is given by

$$\hat{\beta}_R\left[\hat{\Delta}^*\right] = \left[p + \hat{\Delta}^*\right]^{-1} P\hat{\beta}_R, \forall \hat{\delta}^*_{Ri} \geq 0 \quad (5.20)$$

We have, $\hat{\beta}_R\left[\hat{\Delta}^*\right] = \hat{\beta}_R$ when $\hat{\delta}^*_{R_i} = 0, \forall i = 1, 2, ..., K$

By considering the linear restrictions Rβ=r, one may obtain

$$E\left[\hat{\beta}_R(\hat{\Delta}^*)\right] = Q^*\beta + Q^*(X^1X)^{-1}R^1\left[R(X^1X)^{-1}R^1\right]^{-1}(r - R\beta) \quad (5.21)$$

$$\text{Where } Q^* = \left[P + \hat{\Delta}^*\right]^{-1} P = \left[X^1X + \hat{\Delta}^*\right]^{-1}(X^1X) \quad (5.22)$$

Thus , $\hat{\beta}_R(\hat{\Delta}^*)$ is always a biased estimator of β, unless

$\hat{\delta}^*_{Ri} = 0, \forall i = 1, 2, ..., K$ and $(r - R\beta) = 0$, a null vector.

The variance covariance matrix of $\hat{\beta}_R(\hat{\Delta}^*)$ is given by

$$Var\left[\hat{\beta}_R(\hat{\Delta}^*)\right] = \sigma^2 Q^*\left[(X^1X)^{-1} - (X^1X)^{-1} R^1\left[R(X^1X)^{-1}R^1\right]^{-1} R(X^1X)^{-1}\right]Q^{*1} \quad (5.23)$$

Also, the MSE of $\hat{\beta}_R(\hat{\Delta}^*)$ is given by

$$MSE\left[\hat{\beta}_R(\hat{\Delta}^*)\right] = \sigma^2 \; tr\left[Q^*LQ^{*1}\right] + \left\{Q^*(X^1X)^{-1}\eta^* - \hat{\Delta}^*\left[X^1X + \hat{\Delta}^*\right]^{-1}\beta\right\}^1 \left\{Q^1(X^1X)^{-1}\eta^* - \hat{\Delta}^*(X^1X + \hat{\Delta}^*)^{-1}\beta\right\} \quad (5.24)$$

Where $\eta^* = R^1\left[R(X^1X)^{-1} R^1\right]^{-1}(r - R\beta)$

If $(r - R\beta) = 0$ then the MSE$\left[\hat{\beta}_R(\hat{\Delta}^*)\right]$ reduces to

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$$MSE\left[\hat{\beta}_R(\hat{\Delta}^*)\right] = \sigma^2 \, tr\left[Q^* L Q^{*1}\right] + \hat{\Delta}^{*2} \beta^1 \left[X^1 X + \hat{\Delta}^*\right]^{-2} \beta \qquad (5.25)$$

To compare the performance of the proposed adaptive general combined restricted ridge regression estimator $\hat{\beta}_R(\hat{\Delta}^*)$ with (i) the Restricted least squares estimator $\hat{\beta}_R$ and (ii) the ordinary ridge regression estimator $\hat{\beta}_R(\Delta)$, one may use the following two criteria based on minimum sampling variance.

Under the assumption that linear restrictions are true i.e., $(r - R\beta) = 0$, it can be easily shown that

$$\text{(i) } Var\left[\hat{\beta}_R\right] - Var\left[\hat{\beta}_R(\hat{\Delta}^*)\right] = \sigma^2\left[L - Q^* L Q^{*1}\right] \geq 0 \qquad (5.26)$$

$$\text{(ii) } \left\{Var\left[\hat{\beta}(\hat{\Delta}^*)\right] - Var\left[\hat{\beta}_R(\hat{\Delta}^*)\right]\right\} \geq 0 \qquad (5.27)$$

Thus the proposed new adaptive general combined restricted ridge regression estimator $\hat{\beta}_R(\hat{\Delta}^*)$ is always superior to (i) the restricted least squares estimator $\hat{\beta}_R$ and (ii) the ordinary ridge regression estimator $\hat{\beta}_R(\hat{\Delta}^*)$ for β by the criterion of sampling variance.

Remarks: one can compare the performance of the proposed new adaptive general combined restrictive ridge regression estimator with the restricted least squares estimator and an ordinary ridge regression estimator of β by means of the criteria of Minimum mean squared error under the two cases of (i) $(r - R\beta) = 0$ and (ii) $(r - R\beta) \neq 0$.

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VI. CONCLUSIONS:

In the present study, an adaptive general ridge regression estimator has been proposed by reducing the general linear regression model to its canonical form. An iterative procedure has been suggested to set the optimum values for characterizing scalars in the ridge regression estimator.

A new combined restricted ridge regression estimator has been discussed by deriving its mean squared error. The performance of the proposed estimator with the restricted least squares estimator and an ordinary least squares estimator has been examined by using the criterion of sampling variance. As an extension, a new adaptive general combined restricted ridge regression estimator has also been proposed in the present study by using an iterative procedure for the solutions of elements of ridge parameter matrix.

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