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Research Paper

A VIEW ON SUBDIVISION OF SOME GRAPHS

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ABSTRACT

A graph G is an ordered pair G(V,E), where V is the set of p vertices and E is the set of q edges. Research on graph theory includes graph domination, matching, coloring, reconstruction, labeling etc. Our researches are mainly concentrating on the labelling of graphs.

Graph labeling is one of the fascinating areas of graph theory with wide ranging applications. Graph labelling was first introduced in 1960's. An enormous body of literature has grown around graph labelling in the last four decades. Labelled graph provide mathematical models for a broad range of applications.

Keywords:

Graph, set, labeling, applications, edges, mathematical models, vertices, etc.,

Introduction

A pair sum labeling of a graph G(V,E) is defined as an injective map g from V(G) to $\{\pm 1, \pm 2 \ldots \pm p\}$ is such that g_e:E(G)→Z-{0} defined by $g_e(uv)=g(u)+g(v)$ is one-one and $g_e(E(G))$ is either of the form $\{\pm k_1 \ \pm k_2 \ldots\ldots \pm k_{q/2}\}$ or $\{\pm k_1 \ \pm k_2 \ldots\ldots k_{q-1/2}\} \cup \{k_{q+1/2}\}$ accordingly q is even or odd.

The graph considered here will be finite, undirected and simple.

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Definition

The graph obtained by subdividing each edge of a graph G is called the subdivision graph of G.

Theorem

$S(L_n)$ is a pair sum graph, where L_n is a ladder on n vertices.

Proof

Let $V(S(L_n))=\{x_i, y_i, z_i, a_j, b_j : 1\le i \le n, 1\le j \le n-1\}$ and

$$E(S(L_n))=\{x_iz_i, z_iy_i : 1\le i \le n\}\cup\{x_ia_i, a_i x_{i+1} y_i b_i, b_iy_{i+1} : 1\le i \le n-1\}.$$

Case 1: n is even.

When $n = 2$, the proof follows from the Theorem 3.3. For $n > 2$,

Define $g: V(S(L_n))\to\{\pm1,\pm2,\pm3, ..., \pm(5n-2)\}$ by

$$g(x_{n/2}) = -1,$$

$$g(x_{n/2+1}) = -3$$

$$g(x_{n/2-i}) = 10i+3, 1 \le i \le \frac{n-2}{2}$$

$$g(x_{n/2+i+1}) = -10i+1, \quad 1\le i \le \frac{n-2}{2}$$

$$g(z_{n/2}) = 5, \; g(z_{n/2+1}) = -5$$

$$g(z_{n/2-i}) = 10i+1, 1 \le i \le \frac{n-2}{2}$$

$$g(z_{n/2+1+i}) = -(10i+1), 1 \le i \le \frac{n-2}{2}$$

$$g(y_{n/2}) = 3,$$

$$g(y_{n/2+1}) = 1$$

$$g(y_{n/2-i}) = 10i-1, 1 \le i \le \frac{n-2}{2}$$

$$g(y_{n/2+1+i}) = -10i-3, 1 \le i \le \frac{n-2}{2}$$

$$g(a_{n/2}) = -2$$

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$g(a_{n/2-i}) = 10i + 5,\ 1 \le i \le \frac{n-2}{2}$

$g(a_{n/2+i}) = -10i + 3, 1 \le i \le \frac{n-2}{2}$

$g(b_{n/2}) = 2$

$g(b_{n/2-i}) = 10i - 3, \quad 1 \le i \le \frac{n-2}{2}$

$g(b_{n/2+i+1}) = -10i - 5,\ 1 \le I \le. \frac{n-2}{2}$

When $n = 4$,

$g_e(E(S(L_n))) = \{3, 4, 5, 8, 10, 16, 20, 24, 28\} \cup$

$\{-3, -4, -5, -8, -10, -16, -20, -24, -28\}.$

For $n > 4$,

$g_e(E(S(L_n))) = g_e(E(S(L_4))) \cup \{(26,36,40,44,48,38),$

$(-26, -36, -40, -44, -48, -38), (46,56,60,64,68,58),$

$(-46, -56, -60, -64, -68, -58), (10n - 34, 10n - 24, 10n - 20,$

$10n - 16, 10n - 12, 10n - 22), (-10n + 34, -10n + 24, -10n + 20,$

$-10n + 16, -10n + 12, -10n + 22)\}.$

Therefore g is a pair sum labeling.

Case 2. n is odd.

Clearly $S(L_1) \cong P_3$ and hence $S(L_n)$ is a pair sum graph by Theorem 3.1For $n > 1$,

Define

$g: V(S(L_n)) \rightarrow \{\pm 1, \pm 2, \ldots, \pm(5n - 2)\}$ by

$g(x_{(n+1)/2}) = 6, \ g(x_{(n-1)/2}) = 12$

$g(x_{(n+3)/2}) = -12, \ g(a_{(n-1)/2}) = -9$

$g(a_{(n+1)/2}) = 3$

$g(x_{(n+3)/2+i}) = 10i + 10, \quad 1 \le i \le \frac{n-3}{2}$

$g(x_{(n-1)/2-i}) = -(10i + 10), \ 1 \le i \le \frac{n-3}{2}$

$g(y_{(n+3)/2+i}) = -(6 + 10i), 1 \le i \le \frac{n-3}{2}$

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$g(y_{(n-1)/2-i}) = 6 + 10i\ ,\ 1 \le i \le \frac{n-3}{2}$

$g\ (z_{(n+3)/2+i}) = -10i + 2,\ 1 \le i \le \frac{n-3}{2}$

$g(z_{(n-1)/2-i}) = 10i - 2,\ 1 \le i \le \frac{n-3}{2}$

$g(y_{(n+1)/2}) = 2,\ \ g(y_{(n-1)/2}) = 10$

$g(y_{(n+3)/2}) = -10,\ g(b_{(n-1)/2}) = -6$

$g(b_{(n+1)/2}) = 4,$

$g(z_{(n+1)/2}) = -4$

$g\ (z_{(n-1)/2}) = 8,\ g(z_{(n+1)/2}) = -8$

$g\ (a_{(n+1)/2+i}) = -(10i + 12\ )\ ,\ 1 \le i \le \frac{n-3}{2}$

$g(a(_{n-1)/2-i}) = 10i + 12,\ 1 \le i \le \frac{n-3}{2}$

$g(b_{(n+1)/2+i}) = -(10i + 4\ )\ ,1 \le i \le \frac{n-3}{2}$

$g\ (b_{(n-1)/2-i}) = 10i + 4,\ 1 \le i \le \frac{n-3}{2}.$

Therefore

$g_e(\ E\ (S\ (\ L_3))) = \{\ 2, 3, 4, 6, 9, 18, 20, -2, -3, -4, -6, -9, -18, -20\}$

and $g_e(E(S\ (\ L_5))) = g_e(\ E\ (S\ (\ L_3)))\ \ \cup \{\ 24, 30, 34, 38, 42, 36, -24, -30, -34, -38, -42, -36\}$

When $n > 5$,

$g_e(E(S\ (\ L_n))) = g_{\ e}(E(\ S\ (\ L_5))) \cup \{(40,50,54,58,62,52), (-40, -50, -54, -58, -62, -52),$

$(60,70,74,78,82,72), (-60, -70, -74, -78, -82, -72), \ldots,$

$(10n - 30, 10n - 20, 10n - 16, 10n - 12, 10n - 8, 10n - 18),$

$(-10n + 30, -10n + 20, -10n + 16, -10n + 12, -10n + 8, -10n + 18)\}.$

Then g is a pair sum labeling.

Theorem

$\boldsymbol{S(C_n K_1)}$is a pair sum graph

Proof

Let $V(S(C_n K_1))=\{x_j:1\leq j\leq 2n\}\cup\{z_j y_j:1\leq j\leq n\}$

$E(S(C_n K_1))=\{x_j x_{j+1}:1\leq j\leq 2n-1\}\cup\{x_{2j-1} z_j:1\leq j\leq n\}\cup\{y_j z_j:1\leq j\leq n\}$.

Case 1. n is even.

Define $g: S(C_n K_1)\rightarrow\{\pm1,\pm2,\ldots,\pm4n\}$ by

$g(x_j)=2j-1,\ 1\leq j\leq n$

$g(x_{n+i})=-(2j-1),\ 1\leq j\leq n$

$g(z_j)=2n-1+2j,\ 1\leq j\leq \frac{n}{2}$

$g(z_{n/2+j})=-2n+1-2j,\ 1\leq j\leq \frac{n}{2}$

$g(y_j)=3n-1-2j,\ 1\leq j\leq \frac{n}{2}$

$g(z_{n/2+j})=-3n+1+2j,\ 1\leq j\leq \frac{n}{2}$

Here $g_e(E)=\{4,8,12,\ldots,(4n-4)\}\cup\{-4,-8,-12,\ldots,-(4n-4)\}$

$\cup\{2n+2,2n+8,2n+14,\ldots,5n-4\}$

$\cup\{-(2n+2),-(2n+8),-(2n+14),\ldots,-(5n-4)\}\cup$

$\{5n+2,5n+6,5n+10,\ldots,7n-2\}\{-(5n+2),-(5n+6),-(5n+10),\ldots,-(7n-2)\}$.

Then g is pair sum labeling.

Case 2. n is odd.

Define $g: V(S(C_n K_1))\rightarrow\{\pm1,\pm2,\ldots,\pm4n\}$ by

$g(x_j)=4n-2j+2,\ 1\leq j\leq n$

$g(x_{n/2+j})=-4n+2j-2,\ 1\leq j\leq n$

$g(z_j)=-n-1+j,\ 1\leq j\leq\lceil \frac{n}{2}\rceil$

$g(z_{\lceil n/2\rceil+j})=n-j,\ 1\leq j\leq\lfloor \frac{n}{2}\rfloor$

$g(y_j)=-2n-2+2j,\ 1\leq j\leq\lceil \frac{n}{2}\rceil$

$g(y_{\lceil n/2\rceil+j})=2n+2-2j,\ 1\leq j\leq\lfloor \frac{n}{2}\rfloor$

Here $g_e(E (S (C_n K_1))) =\{8n-2, 8n-6, \; 4n+10, 40+6\}$

$\cup \{-(8n-2), -(8n-6), \;, -(4n+10), -(4n+6)\}$

$\cup \{2n-2, -2n+2\} \cup \{3n, 3n-3, 3n-6, \;, 3(n+1)/2\}$

$\cup \{-3n, -(3n-3), -(3n-6), \;, -3(n+1)/2\}$

$\cup \{3n-1, 3n-4, \;, 3(n+7)/2\}$

$\cup \{-(3n-1), -(3n-4), \;, -3(n+7)/2\}$.

Then g is pair sum labeling.

Theorem

$S(P_n K_1)$ is a pair sum graph.

Proof

Let $V(S(P_n K_1)) = \{x_j : 1 \leq j \leq 2n-1\} \cup \{z_j, y_j, : 1 \leq j \leq n\}$

$E(S(P_n K_1))=\{x_j x_{j+1}:1\leq j \leq 2n-2\} \cup \{x_{2j-1} z_j:1\leq j \leq n\} \cup \{y_j z_j:1\leq j \leq n\}$.

Case 1. n is even.

When $n = 2$, the proof follows from Theorem 3.1 For $n > 2$, Define

$g:V(S(P_n K_1)) \to \{\pm1, \pm2, \ldots, \pm(4n-1)\}$ by

$g(x_n)=1$, $g(x_{n-1})= -2$, $g(x_{n+1})=2$

$g(x_{n-1-2j})= -5j-2$, $1\leq j \leq \frac{n}{2}-1$

$g(x_{n-2j})= -(5j+3)$, $1\leq j \leq \frac{n}{2}-1$

$g(x_{n+1+2j})= 5j+3$, $1 \leq j \leq \frac{n}{2}-1$

$g(x_{n+2j})= 5j+2$, $1 \leq j \leq \frac{n}{2}-1$

$g(z_{n/2})= 4$, $g(z_{n/2+1})= -5$

$g(z_{n/2-j})= -5j-4$, $1 \leq j \leq \frac{n-2}{2}$

$g(z_{n/2+j+1})= 5j+4$, $1 \leq j \leq \frac{n-2}{2}$

$g(y_{n/2})= -6$, $g(y_{n/2+1}) = 6$

$g(y_{n/2-j})= -5j-5$, $1 \leq j \leq \frac{n-2}{2}$

$g\,(y_{n/2+j+1}) = 5j + 5, 1 \le j \le \frac{n-2}{2}$

Here

$g_e(E\,(S\,(P_4\,K_1))) = \{1, 2, 3, 9, 15, 17, 19\} \cup \{-1, -2, -3, -9, -15, -17, -19\}$.

For $n > 4$,

$g_e(E(S\,(P_n\,K_1))) = g_e(E\,(S\,(P_4\,K_1))) \cup \{20, 25, 27, 29\} \cup \{-20, -25, -27, -29\}$

$\cup \{30, 35, 37, 39\} \cup \{-30, -35, -37, -39\}$,

$\cup \{5n-10, 5n-5, 5n-3, 5n-1\}$

$\cup \{-(5n-10), -(5n-5), -(5n-3), -(5n-1)\}$.

Then g is pair sum labeling.

Case 2. n is odd.

Since $S(P_1K_1) \cong P_3$, which is a pair sum graph by Theorem 3.2

For $n > 1$, Define $g: V\,(S\,(P_n\,K_1)) \rightarrow \{\pm1, \pm2, ..., \pm(4n-1)\}$ by

$g(x_{(n+1)/2}) = 1,$

$g(x_{(n-1)/2}) = 8$

$g\,(x_{(n+3)/2}) = -8$

$g\,(x_{(n-1)/2-2j}) = -10j + 1,\ 1 \le j \le \lfloor \frac{n}{2} \rfloor - 1$

$g(x_{(n+1)/2-2j}) = 5j + 5,\ 1 \le j \le \lfloor \frac{n}{2} \rfloor - 1$

$g(x_{(n+3)/2+2j}) = -10j - 1,\ 1 \le j \le \lfloor \frac{n}{2} \rfloor - 1$

$g\,(x_{(n+1)/2+2j}) = -(5j + 5),\ 1 \le j \le \lfloor \frac{n}{2} \rfloor - 1$

$g(z_{(n+1)/2}) = -2,\ g(z_{(n-1)/2}) = -5$

$g\,(z_{(n+3)/2}) = 6$

$g(z_{(n-1)/2-j}) = 5j + 7, 1 \le i \le \lfloor \frac{n}{2} \rfloor - 1$

$g\,(z_{(n+3)/2+j}) = -(5j + 7),\ 1 \le j \le \lfloor \frac{n}{2} \rfloor - 1$

$g(y_{(n+1)/2}) = 3,\ g(y_{(n-1)/2}) = 9$

$g(y_{(n+3)/2}) = -9$

$g(y_{(n-1)/2-j}) = 5j + 8,\ 1 \le j \le \lfloor \frac{n}{2} \rfloor - 1$

$g(y_{(n+3)/2+j}) = -(5j + 8),\ 1 \le j \le \lfloor \frac{n}{2} \rfloor - 1$

Here $g_e(E(S(P_3 K_1))) = \{1,-1,2,-2,3,-3,4,-4,5,-5\}$

$g_e(E(S(P_5 K_1))) = g_e(E(S(P_3 K_1))) \cup \{18,21,23,25,-18,-21,-23,-25\}$

When $n > 5$,

$g_e(E(S(P_n K_1))) = \{26,31,33,35\} \cup \{-26,-31,-33,-35\} \cup \{36,41,43,45\}$
$\cup \{-36,-41,-43,-45\} \cup, \ldots, \cup \{5n-9, 5n-4, 5n-2, 5n\}$
$\cup \{-(5n-9), -(5n-4), -(5n-2), -5n\}$.

Then g is a pair sum labeling.

Illustration of theorem is shown in figure

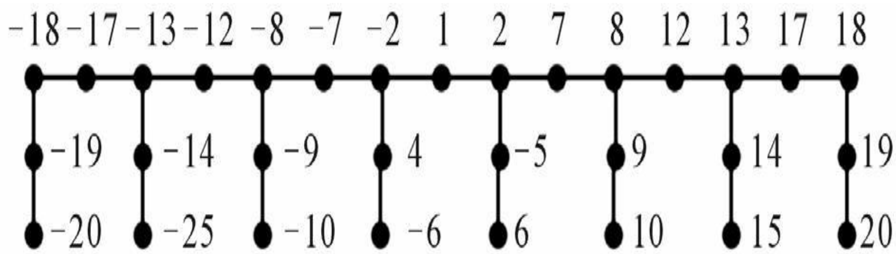

Figure: $S(P_8K_1)$

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