

A STUDY OF LINEAR STABILITY ANALYSIS IN A GRAVITY MODULATION ON DOUBLE DIFFUSIVE CONVECTION IN A FLUID AND POROUS LAYER

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ABSTRACT

A study of linear stability analysis of convection in a binary fluid and saturated porous layer subjected to gravity modulation is performed. Regular perturbation method based on small amplitude of modulation is employed to compute onset threshold for synchronous mode, as a function of frequency.

Keywords: Double diffusive convection, Gravity modulation, Fluid layer, Porous layer, Perturbation method.

1. INTRODUCTION

The problem of DDC in fluid and saturated porous media has received significant interest during the past few decades because of its wide spread applications, such as convective heat and mass transfer, solidification of binary mixtures to the migration of solutes in water-saturated soils and the migration of moisture through air contained in fibrous insulation and so on. Some of the areas where DDC finds exhaustive applications include oceanography, astrophysics, geophysics, geology, chemistry, and metallurgy. The problem of DDC in fluid and porous media has been extensively investigated both theoretically and experimentally and the exhaustive research of the same is well reported by Turner [1-3], Huppert and Turner [4], Platten and Lagros [5], Ingham and Pop [6, 7], Nield and Bejan [8], Vafai [9, 10] and Vadasz [11].

The study of the effect of gravity modulation on the onset of convection in a porous medium is of comparatively recent origin. Although the study of DDC in both fluid and porous media is exhaustively investigated by many researchers, a comparatively little attention has been given to its study under the influence of gravity modulation. The main objective of this article is to analyze the effect of small amplitude gravity modulation on the onset of a binary fluid layer and a saturated porous layer for a wide range of values of frequency of the modulation, solute Rayleigh number, Lewis number, Prandtl number, Darcy number, normalized porosity and viscosity ratio. We intend to provide a fundamental understanding of how the governing parameters would influence natural convection arising from gravity perturbation. As a first attempt, we present a linear stability analysis of a heated fluid layer and a saturated porous layer to explore the effect of various parameters on the onset of DDC in the presence of gravity modulation. In the case of porous layer, both the Darcy and Brinkman models are considered.

2. MATHEMATICAL FORMULATION

We consider an infinite horizontal binary fluid layer / saturated porous layer confined between the planes $z = 0$ and $z = d$ subjected to time-periodically varying gravity force

$\mathbf{g} \equiv (0, 0, -g(t))$ acting on it, where $g(t) = g_0(1 + \varepsilon \cos \bar{\omega}t)$ with g_0 the constant gravity in an otherwise unmodulated system, ε the small amplitude of modulation, $\bar{\omega}$ the frequency and t the time. The temperatures T_l and T_u with $T_l > T_u$ and solute concentrations S_l and S_u with $S_l > S_u$ are imposed at the bottom and top boundaries respectively. A Cartesian frame of reference is chosen with the origin in the lower boundary and z -axis vertically upwards. The interaction between heat and mass transfer, known as Soret and Dufour effects, is supposed to have no influence on the convective flow, so they are ignored. The porous medium is assumed to be isotropic and is in local thermal equilibrium with fluid phase.

The linearized equations governing the perturbations in the form,

$$\nabla \cdot \mathbf{q}' = 0, \quad (1)$$

$$\frac{1}{\phi} \frac{\partial \mathbf{q}'}{\partial t} + \frac{1}{\rho_0} \nabla p' - (\beta_T T' + \beta_S S') g_0 (1 + \varepsilon \cos \bar{\omega}t) \mathbf{k} = A_1 \frac{\mu_e}{\rho_0} \nabla^2 \mathbf{q}' - A_2 \frac{\mu}{\rho_0 K} \mathbf{q}', \quad (2)$$

$$\gamma \frac{\partial T'}{\partial t} - w' \frac{\Delta T}{d} = \kappa_T \nabla^2 T', \quad (3)$$

$$\frac{\partial S'}{\partial t} - \frac{1}{\phi} w' \frac{\Delta S}{d} = \kappa_S \nabla^2 S'. \quad (4)$$

Here \mathbf{k} denotes the unit vector in the z -direction and $\kappa_T = k/(\rho_0 c)_f$ the thermal diffusivity with appropriate definition for fluid layer and porous layer. For the clear fluid layer and Brinkman porous medium, the boundaries are assumed to be stress-free, isothermal and isohaline. Accordingly, the boundary conditions at $z = 0$ and $z = d$ are

$$w' = \frac{\partial^2 w'}{\partial z^2} = T' = S' = 0. \quad (5a) \quad \text{For}$$

Darcy porous medium case, the boundaries are impermeable, isothermal and isohaline. Therefore, the boundary conditions at $z = 0$ and $z = d$ are

$$w' = T' = S' = 0. \quad (5b) \quad \text{By}$$

operating curl twice on Eq. (2) we eliminate p' from it, and then render the resulting equation and the Eqs. (1)-(4) dimensionless using the following transformations

$$(x', y', z') = d(x^*, y^*, z^*), \quad t' = (d^2 / \kappa_T) t^*, \quad (u', v', w') = (\phi \kappa_T / d)(u^*, v^*, w^*), \quad (6) \text{ to}$$

$$p' = (\mu \kappa_T \phi / d^2) p^*, \quad T' = (\Delta T) T^*, \quad S' = (\Delta S) S^*,$$

obtain non-dimensional equations as (on dropping the asterisks for simplicity),

$$\left(\frac{1}{Pr} \frac{\partial}{\partial t} - MA_1 \nabla^2 + A_2 Da^{-1} \right) \nabla^2 w = (1 + \varepsilon \cos \omega t) \nabla_1^2 (Ra_T T - Ra_S S), \quad (7)$$

$$\left(\frac{1}{\chi} \frac{\partial}{\partial t} - \nabla^2 \right) T = w, \quad (8)$$

$$\left(\frac{\partial}{\partial t} - Le^{-1} \nabla^2 \right) S = w, \quad (9)$$

where $Pr = \phi\mu / \rho_0\kappa$, the Prandtl number, $Ra_T = \rho_0\beta_T g\Delta T d^3 / \mu\kappa_T$, the thermal Rayleigh number, $Ra_S = \rho_0\beta_S g\Delta S d^3 / \mu\kappa_T$, the solute Rayleigh number, $Da = K/d^2$, the Darcy number, $\omega = \bar{\omega}d^2/\kappa_T$, the nondimensional frequency of modulation, $Le = \kappa_T/\kappa_S$, the Lewis number, $M = \mu_e/\mu$, the ratio of effective viscosity and fluid viscosity, and $\chi = \phi/\gamma$, the normalized porosity.

The boundary conditions (5a,b) in the non-dimensional form are given by

$$w = \partial^2 w / \partial z^2 = T = S = 0 \quad \text{at } z = 0, 1 \quad \text{and} \quad (10a) \quad w = T = S = 0$$

$$\text{at } z = 0, 1 \quad (10b)$$

After eliminating the coupling between the equations (7)-(9) we obtain the single equation for vertical component of velocity in the form

$$\left[\left(\frac{1}{Pr} \frac{\partial}{\partial t} - MA_1 \nabla^2 + A_2 Da^{-1} \right) \left(\frac{1}{\chi} \frac{\partial}{\partial t} - \nabla^2 \right) \left(\frac{\partial}{\partial t} - Le^{-1} \nabla^2 \right) \nabla^2 \right. \\ \left. - Ra_T \left(\frac{\partial}{\partial t} - Le^{-1} \nabla^2 \right) (1 + \varepsilon \cos \omega t) \nabla_1^2 + Ra_S \left(\frac{1}{\chi} \frac{\partial}{\partial t} - \nabla^2 \right) (1 + \varepsilon \cos \omega t) \nabla_1^2 \right] w = 0. \quad (11)$$

The boundary conditions (10a,b) in terms of the vertical component of velocity become

$$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial^4 w}{\partial z^4} = \frac{\partial^6 w}{\partial z^6} = \frac{\partial^8 w}{\partial z^8} = 0, \quad \text{at } z = 0, 1. \quad (12) \quad \text{Now the}$$

disturbances in the normal modes can be expressed as

$$w = W(z, t) e^{i(lx + my) + \sigma t} \quad (13) \quad \text{where}$$

$W(z, t)$ is a periodic function of time with the same period as the gravity modulation, l, m are the wavenumbers of the disturbances in the x, y directions, respectively, $\sigma = \sigma_r + i\sigma_i$ is the growth rate of the disturbances. Let σ^* be the eigenvalue with greatest real part. The basic state, with respect to the infinitesimal disturbances, is unstable if the real part σ_r^* is greater than zero or stable if σ_r^* is less than zero. Here, unstable means that a disturbance experiences net growth over each modulation cycle, or grows during part of the cycle, but ultimately decays, while stable means that every disturbance decay at every instant. At the neutral stable state σ_r^* is zero. If the imaginary part σ_i^* is also zero simultaneously, the disturbance is synchronous with the periodic basic state. We consider in the present paper only synchronous mode.

Substituting the normal modes (13) into the disturbance equation (11), we obtain

$$\left[\left(\frac{1}{Pr} \frac{\partial}{\partial t} - MA_1(D^2 - a^2) + A_2 Da^{-1} \right) \left(\frac{1}{\chi} \frac{\partial}{\partial t} - (D^2 - a^2) \right) \left(\frac{\partial}{\partial t} - Le^{-1}(D^2 - a^2) \right) (D^2 - a^2) \right. \\ \left. + \left\{ Ra_T \left(\frac{\partial}{\partial t} - Le^{-1}(D^2 - a^2) \right) - Ra_S \left(\frac{1}{\chi} \frac{\partial}{\partial t} - (D^2 - a^2) \right) \right\} (1 + \varepsilon \cos \omega t) a^2 \right] W = 0. \quad (14) \quad \text{with } D \equiv \frac{\partial}{\partial z}$$

and $a^2 = l^2 + m^2$. The associated boundary conditions are

$$W = D^2 W = D^4 W = D^6 W = D^8 W = 0 \quad \text{at } z = 0, 1. \quad (15)$$

Equations (14) with boundary conditions (15), is homogeneous system and thus constitute an eigenvalue problem.

3. METHOD OF SOLUTION

We seek the eigenfunctions W and the eigenvalues Ra_T associated with the system of Eqs. (14)-(15) for a modulated gravity field that is different from the constant gravity field by a small quantity of order ε . The eigenfunction W and eigenvalue Ra_T should be a function of ε and they should be obtained for a given Darcy number Da , Prandtl number Pr , solute Rayleigh number Ra_S , Lewis number Le and frequency ω . Since ε is very small for the problem under consideration, we expand these eigenfunctions and eigenvalues in a power series of ε in accordance with the theory of small parameter perturbation, in the form

$$(W, Ra_T) = (W_0, R_0) + \varepsilon(W_1, R_1) + \varepsilon^2(W_2, R_2) + \dots$$

(16) Here W_0 and R_0 are the eigenfunctions and eigenvalues respectively of the unmodulated system and W_i and R_i , ($i \geq 1$) are the corrections to W_0 and R_0 in the presence of gravity modulation.

Substituting Eq. (16) into Eq. (14) and equating the corresponding terms, we obtain the following system of equations

$$LW_0 = 0, \quad (17)$$

$$LW_1 = -a^2 R_0 L_3 \cos \omega t W_0 - a^2 R_1 L_3 \sin \pi z + a^2 Ra_S L_2 \cos \omega t W_0, \quad (18)$$

$$LW_2 = -a^2 R_1 L_3 \cos \omega t W_0 - a^2 R_2 L_3 W_0 - a^2 R_0 L_3 \cos \omega t W_1 - a^2 R_1 L_3 W_1 \\ + a^2 Ra_S L_2 \cos \omega t W_1, \quad (19)$$

Where the operators L and L_i 's are as given in Appendix through Eqs. (A.1)-(A.4).

Each of W_n are required to satisfy the boundary conditions (15). Equation (17) which is obtained at $O(\varepsilon^0)$ is the one used in the study of thermal convection in a horizontal fluid layer/ fluid-saturated porous layer subject to the constant gravitational field. The marginally stable solutions for that problem are

$$W_0^{(n)} = \sin n \pi z, \quad (20) \quad \text{with}$$

the corresponding eigenvalues

$$R_0^{(n)} = MA_1 \frac{(n^2 \pi^2 + a^2)^3}{a^2} + A_2 Da^{-1} \frac{(n^2 \pi^2 + a^2)^2}{a^2} + Ra_S Le. \quad (21) \text{ For a fixed}$$

wavenumber the least eigenvalue occurs for $n = 1$, and is given by

$$R_0 = MA_1 \frac{(\pi^2 + a^2)^3}{a^2} + A_2 \frac{(\pi^2 + a^2)^2}{a^2} Da^{-1} + Ra_S Le, \quad (22) \text{ corresponding}$$

to $W_0 = \sin \pi z$.

Fluid layer

For a fluid layer, $A_1 = 1$, $A_2 = 0$ and then Eq. (32) yields

$$R_0 = \frac{(\pi^2 + a^2)^3}{a^2} + Ra_S Le, \quad (23) \text{ which}$$

assumes the minimum value R_{0c} for $a = a_c$, where a_c satisfies the equation

$$3a_c^2 (\pi^2 + a_c^2)^2 - (\pi^2 + a_c^2)^3 = 0. \quad (24) \text{ We}$$

observe that Eqs. (23) and (24) are the classical results obtained for DDC in binary fluid layer without gravity modulation (see e.g., Turner [1]). Further, these equations yield the values $R_{0c} = 27\pi^4/4 = 657.5$ and $a_c = \pi/\sqrt{2}$, which are associated with the classical Rayleigh-Benard problem.

Darcy model

For Darcy porous medium, i.e. for a densely packed porous layer, $A_1 = 0$, $A_2 = 1$, then Eq. (32) reads

$$R_0 = \frac{(\pi^2 + a^2)^2}{a^2} Da^{-1} + Ra_S Le. \quad (25) \text{ When both}$$

sides of Eq. (35) are multiplied by Da one can obtain the expression for Darcy-Rayleigh number

$$R_{D0} = \frac{(\pi^2 + a^2)^2}{a^2} + Ra_{SD} Le, \quad (26) \text{ with}$$

modified solute Rayleigh number, $Ra_{SD} = \rho_0 \beta_s g \Delta S d K / \mu \kappa_T$ for the Darcy porous layer. The corresponding critical values of Darcy-Rayleigh number and wavenumber are, respectively, given by

$$R_{Doc} = 4\pi^2 \text{ and } a_c = \pi. \quad (27)$$

RESULTS AND DISCUSSION

The onset of binary convection in a horizontal fluid layer and fluid saturated porous layer, under the influence of time-periodically varying gravitational force, is investigated analytically using the linear stability theory. Due to gravity modulation there is a shift in the onset criteria. The critical Rayleigh number and the wavenumber are computed using regular perturbation technique based on the assumption that amplitude of imposed modulation is very small. Because of this we have restricted our analysis only for the first order correction to the critical Rayleigh number, viz., R_{2c} . The critical correction Rayleigh number is evaluated as a function of the frequency of modulation ω , the solute Rayleigh number Ra_s , the Lewis number Le , the Prandtl number Pr , the Darcy number Da , the viscosity ratio M and the normalized porosity χ . The influence of these governing parameters on the onset of DDC is displayed through the Figs. 1-14.

The value of frequency of modulation plays an important role in validating the results obtained in this analysis. When ω is very small the period of modulation becomes sufficiently large and the disturbances grow to a large extent and therefore, the entire system under consideration becomes unstable. This is justified by the magnitude of R_{2c} , which is found to be sufficiently small or even negative in some cases. On the other-hand when ω is very large the effect of gravity modulation is confined only to a narrow boundary layer near the boundary. This is due to the fact that the high frequencies correspond to renormalization of the static gravity field. Thus, outside this thickness the buoyancy force takes a mean value tending towards the equilibrium state value of the unmodulated case. The effect of gravity modulation is therefore significant only for the moderate values of ω . Further, due to the assumption that the amplitude of modulation is small and Darcy resistance dampens the convection currents, the nonlinear effects may be neglected.

In Figs. 1-3 the variation of critical correction Rayleigh number R_{2c} with the frequency of modulation ω is revealed, for the case of fluid layer. It is observed that R_{2c} is negative for small ω while for moderate values of ω , there is a considerable increase in the value of R_{2c} . Thus, the low frequency gravity modulation destabilizes the system where as the convection is delayed when ω is quite large. The system becomes most stable when R_{2c} attains a maximum value corresponding to a specific frequency $\omega = \omega^*$. If ω is increased beyond ω^* we notice that R_{2c} goes on decreasing and becomes independent of ω for large values of frequency. Thus, critical Rayleigh number tends to its equilibrium value of unmodulated state.

The variation of R_{2c} with ω for different values of Ra_s is displayed in Fig. 1. When $Ra_s = 0$ a curve similar to that of a single component case is obtained. In this case R_{2c} is positive over the entire range of values of ω . This indicates the stabilizing effect of gravity modulation on the onset of thermal convection in a viscous fluid layer. However, when $Ra_s \neq 0$, R_{2c} is negative for small values of ω . Thus the presence of second diffusing agent namely the solute concentration leads the gravity modulation to advance the convection as compared to the unmodulated case. For moderate frequency the stabilizing effect is noticed

and at $\omega = \omega^*$, the system becomes most stable due to both gravity modulation and solute gradient. Further it is found that ω^* increase with Ra_s .

In Fig. 2 the effect of Le on the stability of the binary fluid layer is exhibited. The role of Le is to stabilize the system. The frequency ω^* at which the system is most stable is independent of Le . When $Le = 0$, we observed that R_{2c} is negatively very small over the entire domain of ω . Thus in this case the gravity modulation shows a very weak destabilizing effect. The Fig. 3 depicts the variation of R_{2c} with ω for different values of Pr . It is reported that the influence Pr is to enhance the stabilizing effect of Ra_s and Le . This figure also indicates that ω^* increases with Pr .

For the case of a densely packed porous layer saturated with a binary fluid the variation of R_{2c} with ω for various governing parameters is exposed through the Figs. 4-8. We detect that R_{2c} is positive over the entire realm of ω . Therefore, the onset of DDC is delayed as compared to the unmodulated system. It is important to note that the range of values of ω over which the effect of gravity modulation is significant is comparatively larger than the cases of viscous fluid layer and the Brinkman porous layer.

In Fig. 4 the influence of Da is revealed. It is found that with the increasing values of Da , there is a decrease in R_{2c} . Further the range of values of frequency for which R_{2c} becomes independent of ω is reduced considerably for the larger Da . Thus when Da is very small the effect of gravity modulation is more pronounced and is sustained for larger range of frequency. It is also noticed from this figure that ω^* decreases with Da . Thus the Da retards the stabilizing effect of gravity modulation.

It is important to note from Fig. 5 that R_{2c} decreases significantly with Ra_s . Thus the increasing solute gradient shifts R_{2c} towards the lower value. Therefore, there is a net decrease in the value of critical Rayleigh number. This shows a destabilizing effect of Ra_s , which is in contrast to the case of onset of DDC in Darcy porous layer in the absence of gravity modulation. This figure also shows that ω^* is almost independent of Ra_s .

5. CONCLUSIONS

The analytical study of the effect of gravity modulation on the onset of DDC in a binary fluid layer and a saturated porous layer is carried out. The critical correction Rayleigh number is computed using regular perturbation method and the variation of the same with frequency of the imposed modulation is shown graphically and the following conclusions are drawn:

- Due to imposed gravity modulation there is a shift in the onset criteria.
- For small ω , R_{2c} is sufficiently small or even negative therefore the low frequency gravity modulation advances the convection in a binary fluid / saturated porous layer.
- When ω is very large the effect of gravity modulation is confined only to a narrow boundary layer. Outside this thickness the buoyancy force takes a mean value tending towards the equilibrium state value of the unmodulated case.
- The effect of gravity modulation is significant only for the moderate values of ω .

- The nonlinear effects are neglected due to the assumption that the amplitude of modulation is small and Darcy resistance dampens the convection currents.
- The system becomes most stable when R_{2c} attains a maximum value corresponding to a specific frequency $\omega = \omega^*$.

For the case of binary fluid layer

- When $Ra_S = 0$, R_{2c} is positive over the entire range of values of ω this is in agreement to the result of single component case.
- When $Ra_S \neq 0$, R_{2c} is negative for small values of ω and for moderate frequency the stabilizing effect is noticed.
- Le and Pr enhance the stabilizing effect of gravity modulation and Ra_S .
- When $Le = 0$, gravity modulation shows a very weak destabilizing effect.
- ω^* increase with Ra_S and Pr while it is independent of Le.

For the case of binary fluid-saturated densely packed porous layer

- R_{2c} is positive over the entire realm of ω , indicating the inhibition of onset of DDC as compared to the unmodulated system.
- The range of values of ω over which the effect of gravity modulation is significant is comparatively larger than the cases of viscous fluid layer and the Brinkman porous layer.
- Da retards while Le, χ and Pr reinforce the stabilizing effect of gravity modulation.
- The range of values of frequency for which R_{2c} becomes independent of ω is reduced considerably for the larger Da.
- When Da is very small the effect of gravity modulation is more pronounced and is sustained for larger range of frequency.
- ω^* decreases with Da, while increases with Pr and it is almost independent of Ra_S , Le and χ .
- R_{2c} decreases significantly with Ra_S . This shows a destabilizing effect of Ra_S , which is in disparity with the case of onset of DDC in Darcy porous layer in the absence of gravity modulation.
- When $Pr > 50$, the effect of gravity modulation is prevailed over a larger range of frequency.

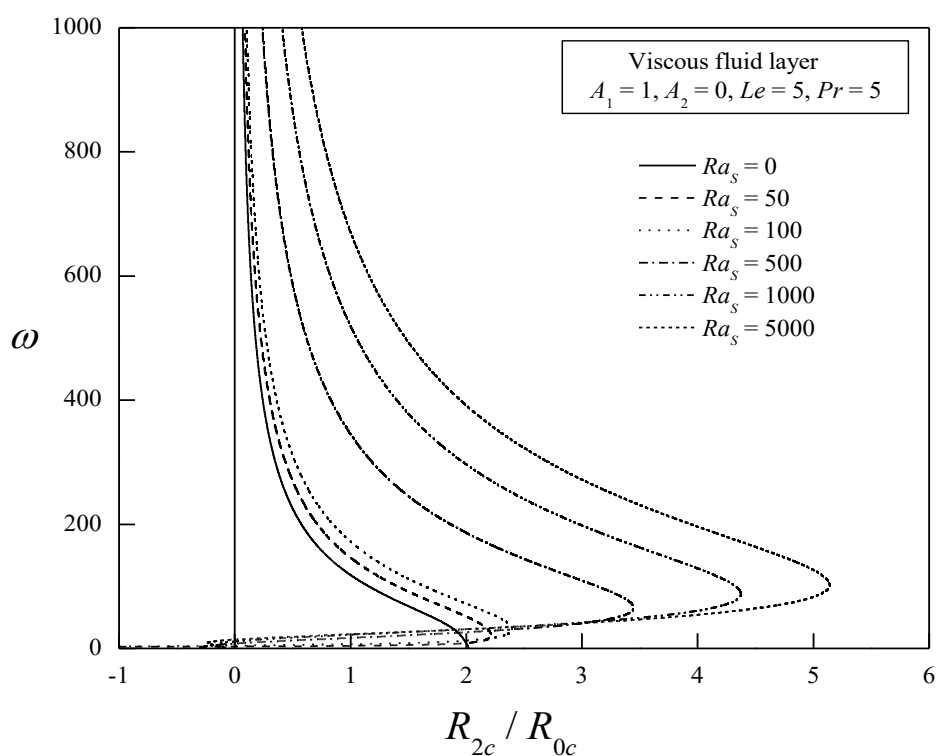


Fig. 1. Variation of R_{2c} with ω for different values of Ra_s for the viscous fluid layer.

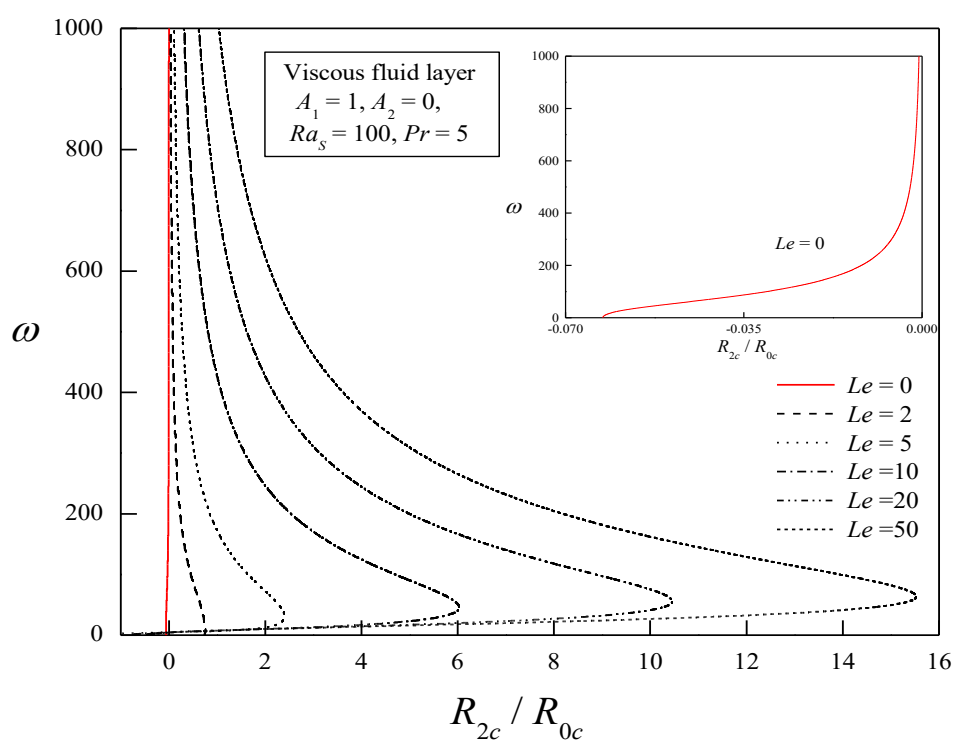


Fig. 2. Variation of R_{2c} with ω for different values of Le for the viscous fluid layer.

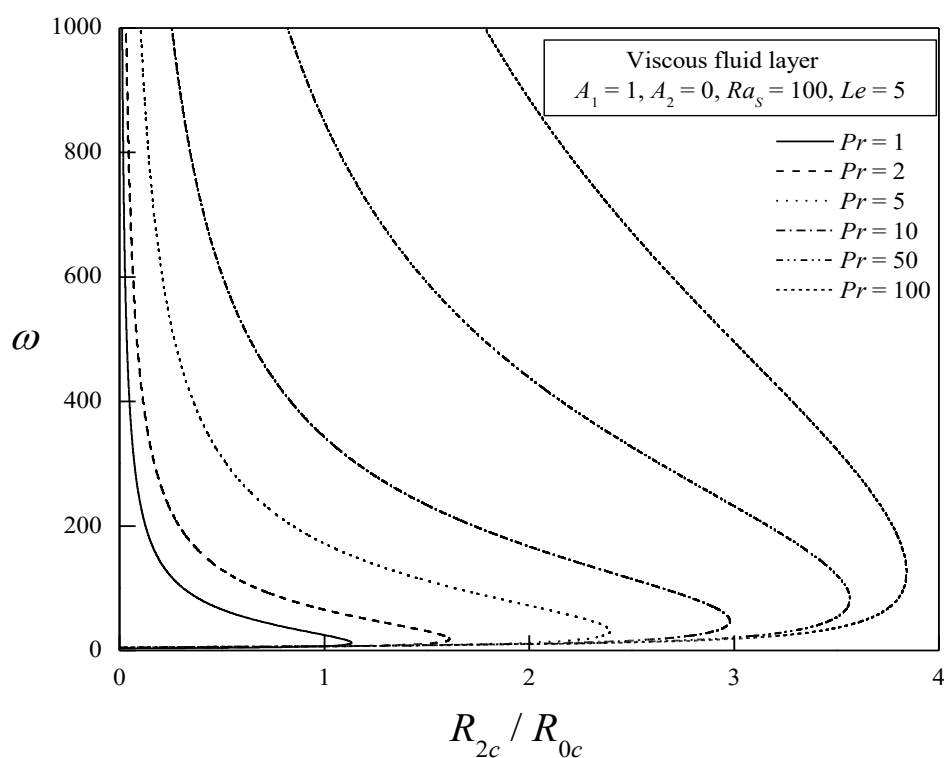


Fig. 3. Variation of R_{2c} with ω for different values of Pr for the viscous fluid layer.

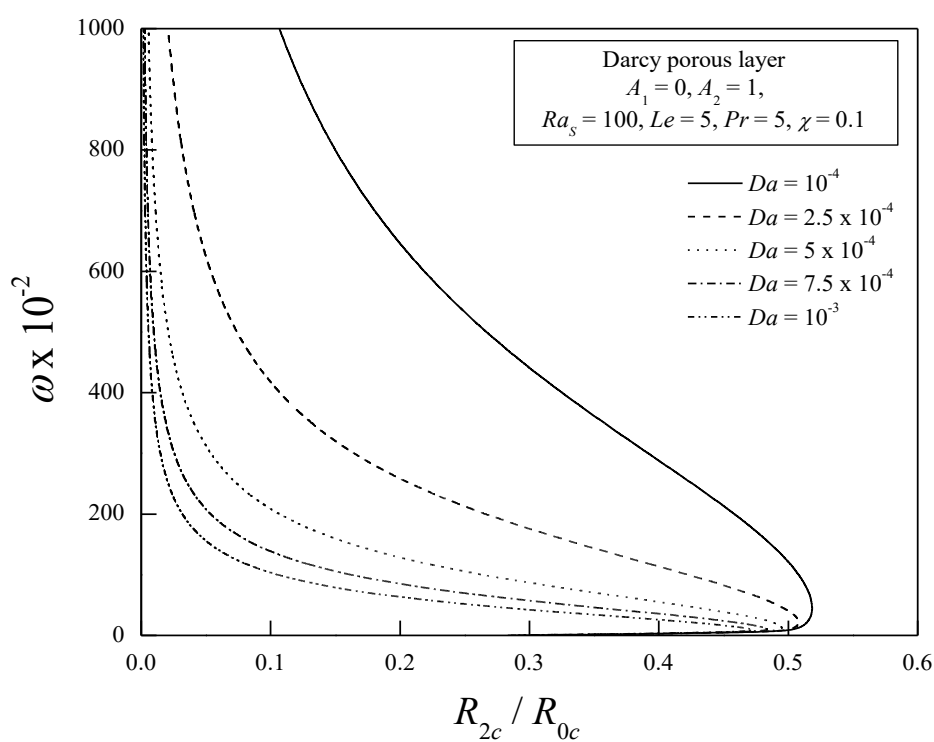


Fig. 4. Variation of R_{2c} with ω for different values of Da for the Darcy porous layer.

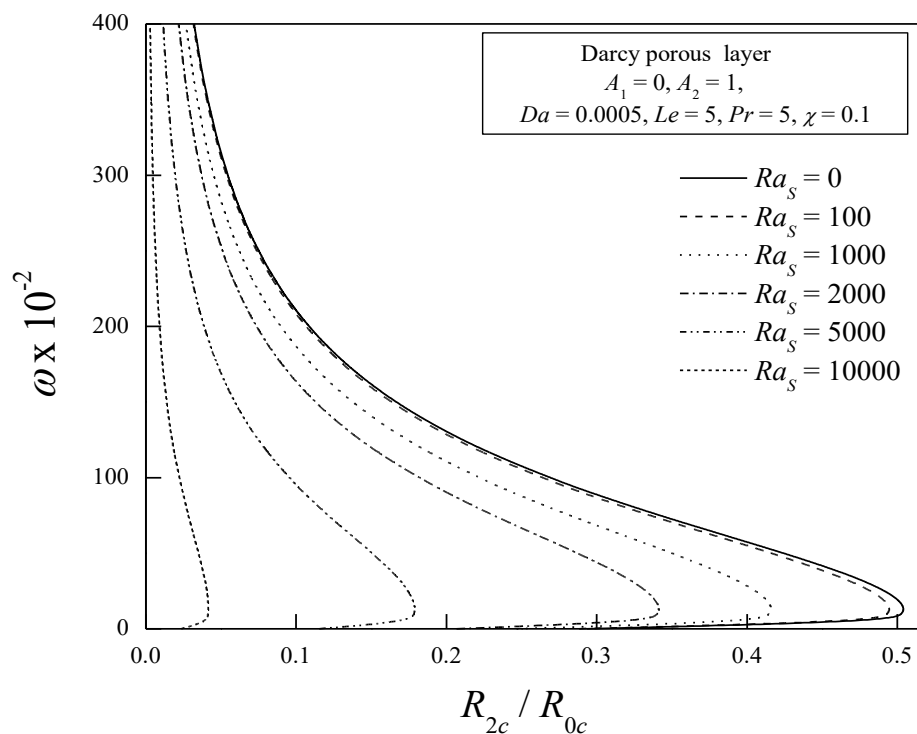


Fig. 5. Variation of R_{2c} with ω for different values of Ra_s for the Darcy porous layer.

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