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NATURAL CONVECTION IN ANISOTROPIC ROTATING POROUS RECTANGULAR CHANNELS USING THERMAL NON-EQUILIBRIUM MODEL

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Abstract

Linear stability of a rotating fluid saturated porous medium heated from below is studied when the fluid and solid phases are not in local thermal equilibrium. The Darcy model which includes Coriolis term with anisotropy permeability is employed as a momentum equation. The critical Rayleigh number for the onset of convection using linear stability analysis is found numerically as a function of mechanical anisotropy parameters, interphase heat transfer coefficient, aspect ratio and Taylor number. It is found that a small interphase heat transfer coefficient has significant effect on the stability of the system. The rotation inhibits the onset convection. The effect of porosity modified conductivity ratio, diffusivity ratio, interphase heat transfer coefficient on the stability of the system is investigated.

Keyword:Porous medium,anisotropy,Taylor number.

Introduction

Thermal convection in fluid saturated porous media is of interest due to its applications in different fields, such as geothermal energy utilization, nuclear waste disposal, biomechanics where fluids flow through lungs and arteries, solar power collectors, design of nuclear reactors and compact heat exchanges. A comprehensive literature survey on this subject can be found in the recent hooks by Ingham and Pop[1], Nield and Bejan [2]. In modeling a fluid-saturated porous medium, most of the investigators assumed a state of local thermal equilibrium (LTE) between the fluid and solid phase at any point in the medium. This is common for most of the studies where the temperature gradient at any location between the two phases is assumed to be negligible. The substantial part of theoretical and experimental works on convective flow in porous media has dealt with isotropic materials. However, in many practical situations the porous materials are anisotropic in their mechanical and thermal properties. Anisotropy is generally consequences of preferential orientation or asymmetric geometry of porous matrix or fibers and is in fact encountered in numerous systems in industry and nature. Anisotropy can also be a characteristic of artificial porous materials like pelletting used in chemical engineering process and fiber material used in insulating purpose. An excellent review of research on convective flow through anisotropic porous media has been given by McKibben [3]. Nilsen and Storesletton [4] presented an analytical study of two dimensional natural convection in horizontal rectangular channels filled with an isotropic/anisotropic porous medium.

Most of the works on convective instability in porous media have been investigated mainly under the assumption that the fluid and porous medium are everywhere in local thermodynamic equilibrium. However, in many practical applications, the solid and fluid phases are not in local thermal equilibrium. Nield and Bejan [S] have discussed a two field model for energy equation. Instead of having a single energy equation, which describes the common temperature of the saturated porous media, two equations are used for fluid and solid phase separately. In a two field model the energy equations are used for fluid and solid phase separately. In a two field model the energy equations are coupled by the terms, which accounts for heat lost or gained from the other phase. Nield [6] has discussed a local thermal non-equilibrium (LTNE) conditions exists due to many obvious causes, such as the presence of distributed or concentrated heat sources in one phase or the presence of some agency which forces different fluid and solid boundary temperature conditions. In fact, LTNE can be ruled out only if steady conduction with uniform solid and fluid thermal conductivities, is the only heat transfer process. As discussed by Banu and Rees [7] when non equilibrium effects are included in the problem the linear analysis is modified and it is still possible to proceed analytically to find the condition for one onset of convection. The stability of a horizontal fluid saturated sparsely packed porous layer heated from below and cooled from above when the solid and fluid phase are not in local thermal equilibrium is examined analytically by Malashetty et al. [8]. The stability of a horizontal fluid saturated anisotropic porous layer heated from below and cooled from above when the solid and fluid phase are not in local thermal equilibrium is examined analytically by Malashetty et al. [9] The rotation effect on thermal convection in porous media has been extensively studied for the case of local thermal equilibrium model. Qin and Kaloni [10] and Govender [11-12] have studied thermal convection in a rotating porous layer using a thermal non-equilibrium model.

In this paper we discuss the onset of convection in an anisotropic rotating porous rectangular channel heated from below when the fluid and solid phases are not in

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local thermal equilibrium. Walls of the channel are non-uniformly heated to establish a linear temperature gradient and they are assumed to be impermeable and perfectly conducting. The critical Rayleigh number using linear stability analysis is obtained numerically as a function of mechanical anisotropy parameters, interphase heat transfer coefficient, Taylor number and aspect ratio.

Mathematical Formulation

Consider two-dimensional free convection in a horizontal rotating porous channel heated from below. The lower surface is held at temperature $T_1 = T_0 + \Delta T$ while upper surface $T_u = T_0$. We assume that the solid and fluid phases of the medium are not in local thermal equilibrium and use a two field model for temperatures with anisotropy in thermal conductivities. The channel is rectangular with height 'h' and width 'a', we choose a cartesian co-ordinate system with z-axis is in the vertical direction and x-axis is the horizontal direction perpendicular to the channel axis. The horizontal channel walls are z = 0 and z = h and the vertical walls at $x = -\frac{a}{2}$ and $x = \frac{a}{2}$. On assuming that the Prandtl-Darcy number is large, so that inertia term may be neglected and invoking Boussinesq approximation, the governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \qquad \ldots(1)$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\nu}{k_x}u + 2\Omega v = 0, \qquad \ldots(2)$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \frac{\nu}{k_y}u - 2\Omega u = 0, \qquad \ldots(3)$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial z} + \frac{\nu}{k_z}w + \frac{\rho}{\rho_0}g = 0, \qquad \ldots(4)$$

$$\varepsilon(\rho c)_f\frac{\partial T_f}{\partial t} + (\rho c)_f(q.\nabla)T_f = \varepsilon K_f\nabla^2 T_f = h\left(T_s - T_f\right), \qquad \ldots(5)$$

$$(1-\varepsilon)(\rho c)_s\frac{\partial T_s}{\partial t} = (1-\varepsilon)k_s\nabla^2 T_s - h\left(T_s - T_f\right), \qquad \ldots(6)$$

$$\rho = \rho_0\left[1 - \beta\left(T_l - T_u\right)\right] \qquad \ldots(7)$$

Since the flow is two-dimensional, we introduce the stream function ψ as:

$$u=\frac{\partial \psi}{\partial z}, w=-\frac{\partial \psi}{\partial x}, \qquad \ldots(8)$$

we also define non-dimensional variables by

$$x=a x^{*}, y=a y^{*}, z=h\, z^{*}, u=\frac{\varepsilon k_{fz} a}{(\rho c)_{f} h^{2}} u^{*},$$

$$v=\frac{\varepsilon k_{fz} a}{(\rho c)_{f} h^{2}} . v^{*}, w=\frac{\varepsilon k_{fz}}{(\rho c)_{f} h} w^{*},$$

$$t=\frac{(\rho c)_{f} h^{2}}{k_{1z}} t^{*}, p=\frac{k_{fz} v \rho_{0}}{(\rho c)_{f} k_{x}} p^{*}, T_{f}=\Delta T\left[T_{0}^{*}+1-z+\theta^{*}\right],$$

$$T_{s}=\Delta T\left[T_{0}^{*}+1-z+\phi^{*}\right], \quad \theta=(\Delta T) \theta^{*}, \phi=(\Delta T) \phi^{*},$$

$$T_{0}=(\Delta T) T_{0}^{*}, \psi=\frac{\varepsilon k_{fz} a}{(\rho c)_{f} h} \psi^{*}.$$

Using the equations (8) and (9) in equations (2)-(7) the non-dimensional equation can be obtained in the form;

$$\frac{\partial p}{\partial x}+\varepsilon \frac{a^{2}}{h^{2}} \frac{\partial \Psi}{\partial z}+\varepsilon \frac{a^{2}}{h^{2}}(T a) v=0, \qquad \ldots(10)$$

$$\frac{\partial p}{\partial y}+\varepsilon \frac{a}{h^{2}} \frac{\kappa_{x}}{\kappa_{y}} v-\varepsilon \frac{a^{2}}{h^{2}}(T a) \frac{\partial \psi}{\partial z}=0, \qquad \ldots(11)$$

$$\frac{\partial p}{\partial z}-R a \theta-\varepsilon \frac{k_{x}}{k_{z}} \frac{\partial \psi}{\partial x}=0, \qquad \ldots(12)$$

$$\eta_{f} \frac{\partial^{2} \theta}{\partial z^{2}}+\frac{\partial^{2} \theta}{\partial z^{2}}-\frac{\partial \psi}{\partial x}=\frac{\partial \theta}{\partial t}+\frac{\partial \Psi}{\partial z} . \frac{\partial \theta}{\partial x}-\frac{\partial \Psi}{\partial x} . \frac{\partial \theta}{\partial z}+H(\theta-\phi). \quad \ldots(13)$$

$$\eta_{1} \frac{\partial^{2} \phi}{\partial x^{2}}+\frac{\partial^{2} \phi}{\partial z^{2}}=\alpha \frac{\partial \phi}{\partial t}-\gamma H(\theta-\phi). \qquad \ldots(14)$$

Due to symmetry about y axis, $\frac{\partial v}{\partial y}=0$ and considering $k_{x}=k_{y}$,

we get equation (11) as

$$v=T a \frac{\partial \psi}{\partial z}.$$

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Eliminating pressure between (10) and (12) we get

$$\xi\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}+Ta\frac{\partial v}{\partial z}+\xi Ra\frac{\partial\theta}{\partial x}=0. \qquad \ldots(16)$$

Differentiating equation (15) w.r.t. z and substituting in (16), we get

$$\xi\frac{\partial^2\psi}{\partial x^2}+\left(1+Ta^2\right)\frac{\partial^2\psi}{\partial z^2}+\xi Ra\frac{\partial\theta}{\partial x}=0. \qquad \ldots(17)$$

In the above equations we have used the following definitions:

$$\xi=\frac{k_s}{k_f}\left(\frac{h}{a}\right)^2, n_f=\frac{k_{fx}}{k_{ft}}\left(\frac{h}{a}\right)^2 .\eta_s=\frac{k_{sx}}{k_{az}}\left(\frac{h}{a}\right)^2, \alpha=\frac{(\rho c)_s}{(\rho c)_f}\frac{k_{fz}}{k_{sz}}$$

$$\gamma=\frac{\varepsilon k_{fz}}{(1-\varepsilon)k_{sz}}, H=\frac{h^3}{\varepsilon k_{fx}}, Ra=\frac{\rho_0 g\beta\Delta T k_z h}{\varepsilon\mu k_{fz}}, Ta=\left(\frac{2\Omega\kappa_x}{v}\right) \qquad \ldots(18)$$

Linear Stability analysis and numerical analysis

The linearised forms of the governing equations (13), (14) and (17) are

$$\xi\frac{\partial^2\psi}{\partial x^2}+\left(1+Ta^2\right)\frac{\partial^2\psi}{\partial z^2}++\xi Ra\frac{\partial\theta}{\partial x}=0 \qquad \ldots(19)$$

$$n_f\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial z^2}-\frac{\partial\psi}{\partial x}=\frac{\partial\theta}{\partial t}+H\left(\theta-\phi\right), \qquad \ldots(20)$$

$$\eta_s\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}=\alpha\frac{\partial\phi}{\partial t}-\gamma H\left(\theta-\phi\right). \qquad \ldots(21)$$

The boundary conditions are

$$\psi=\theta=\phi=0 \text{ at } \left\{\begin{array}{ll} x=-\frac{1}{2}, x=\frac{1}{2}, & 0<z<1 \\ z=0, z=1, & -\frac{1}{2}<x<\frac{1}{2}\end{array}\right\} \qquad \ldots(22)$$

The onset of stationary convection is described by the linear version of equations (19) - (21) and the solution for ψ, θ and ϕ is now taken as a single-mode component as:

$$\psi=D(x)\sin\pi z, \theta=G(x)\sin\pi z, \phi=I(x)\sin\pi z \qquad \ldots(23)$$

In terms of D, G and I the boundary conditions are

$$D\left(\pm\frac{1}{2}\right)=0, G\left(\pm\frac{1}{2}\right)=0, I\left(\pm\frac{1}{2}\right)=0. \quad \ldots(24)$$

Using (23)-(24) in equations (19)-(21), we get

$$\left(\xi\frac{d^2}{dx^2}-\left(1+Ta^2\right)\pi^2\right)D(x)=\xi R_a G(x)=0, \quad \ldots(25)$$

$$\left(\eta_f\frac{d^2}{dx^2}-\pi^2\right)G(x)-\frac{dD(x)}{dx}=\sigma G(x)+H(G-I), \quad \ldots(26)$$

$$\left(\eta_s\frac{d^2}{dx^2}-\pi^2\right)I(x)=\alpha\sigma I(x)-\gamma H(G-I). \quad \ldots(27)$$

Anisotropic case: $\xi\neq\eta_f\neq\eta_s$

By eliminating $D(x)$ and $I(x)$ between (27) – (29), we get a sixth order differential equation in the form:

$$[\left(\eta_f\eta_x\xi\right)D^6-\left(\left(H+\pi^2-Ra\right)\eta_s\xi+\eta_f\left(H\gamma\xi+\pi^2\left(\eta_s+\eta_s Ta^2+\xi\right)\right)\right)D^4+$$
$$\left(\begin{array}{l}\pi^4\eta_s+\eta_s\pi^4 Ta^2+\eta_f\pi^2\left(1+Ta^2\right)\left(\pi^2+H\gamma\right)+\pi^4\xi-\pi^2 Ra\xi+\\ H\left(-Ra\gamma\xi+\pi^2\left(\eta_s+\eta_s Ta^2+\xi+\gamma\xi\right)\right)\end{array}\right)D^2 \quad \ldots(28)$$
$$-\pi^4\left(1+Ta^2\right)\left(H+\pi^2+H\gamma\right)]G(x)=0, \quad \left(D=\frac{d}{dx}\right),$$

with boundary conditions

$$G\left(\pm\frac{1}{2}\right)=G'\left(\pm\frac{1}{2}\right)=G''\left(\pm\frac{1}{2}\right)=0 \quad \ldots(29)$$

The general solution of equation (28) is

$$G(x)=c_1e^{m_1x}+c_2e^{m_2x}+c_3e^{m_3x}+c_4e^{m_4x}+c_5e^{m_5x}+c_6e^{m_6x}, \quad \ldots(30)$$

where $c_i's$ are arbitrary constants and m_i are roots of the auxiliary equation of (28). Since the auxiliary equation involves cubic in D^2, put $m_2=-m_1, m_4=-m_3$, $m_6=-m_5$, where

$$m_1 = \frac{1}{6\delta_1^2}\left[2\delta_2^2 + \left(2^{\frac{4}{3}}\frac{K_1}{K_3}\right) + \left(2^{\frac{2}{3}}K_3\right)\right], \qquad \ldots(31)$$

$$m_3 = \frac{1}{12\delta_1^2}\left[4\delta_2^2 - \left(2^{\frac{4}{3}}\left(1+\sqrt{-3}\right)\frac{K_1}{K_3}\right) + \left(2^{\frac{2}{3}}\left(-1+\sqrt{-3}\right)K_3\right)\right] \qquad \ldots(32)$$

$$m_5 = \frac{1}{12\delta_1^2}\left[4\delta_2^2 + \left(2^{\frac{4}{3}}\left(-1+\sqrt{-3}\right)\frac{K_1}{K_3}\right) - \left(2^{\frac{2}{3}}\left(1+\sqrt{-3}\right)K_3\right)\right]. \qquad (33)$$

$$\delta_1^2 = \eta_f \eta_s \xi,$$

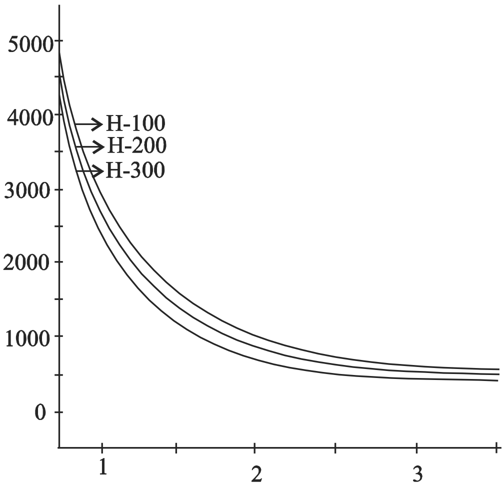
$$\delta_2^2 = \left(\left(H+\pi^2-Ra\right)\eta_s\xi + \eta_f\left(H\gamma\xi + \pi^2\left(\eta_s + \eta_s T^2 a + \xi\right)\right)\right)$$

$$\delta_3^2 = \left(\begin{array}{l}\pi^4\eta_s + \eta_s\pi^4 T^2 a + \eta_f\pi^2\left(1+T^2a\right)\left(\pi^2+H\gamma\right) + \pi^4\xi - \pi^2 Ra\xi + \\ H\left(-Ra\gamma\xi + \pi^2\left(\eta_s + \eta_s T^2 a + \xi + \gamma\xi\right)\right)\end{array}\right) \qquad (34)$$

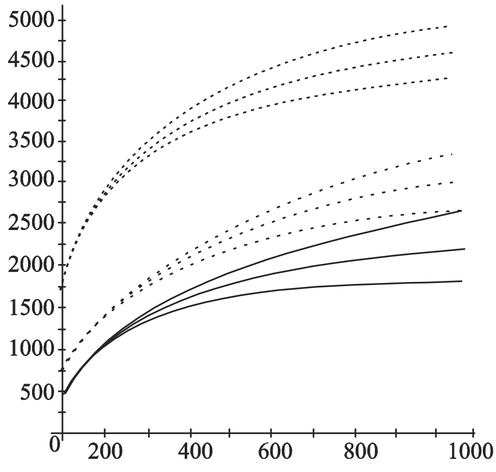
$$\delta_4^2 = \pi^4\left(1+T^2a\right)\left(H+\pi^2+H\gamma\right), \qquad K_1 = \delta_2^4 - 3\delta_1^2\delta_3^2,$$

$$K_2 = 2\delta_2^6 - 9\delta_1^2\delta_2^2\delta_3^2 + 27\delta_1^4\delta_4^2, K_3 = \left(K_2 + \sqrt{-4\left(K_1\right)^3 + \left(K_2\right)^2}\right)^{\frac{1}{3}}$$

The solution of equation (28) using (29) can be obtained using Newton-Raphson method, for various values of $\xi, \eta_f, \eta_s, H, Ta$ and Ra_c can be calculated numerically.



Variation of critical Rayleigh number R_{ac} with ζ for various values of H.



Variation of critical Rayleigh number Ra_c with H for anisotropic parameter γ

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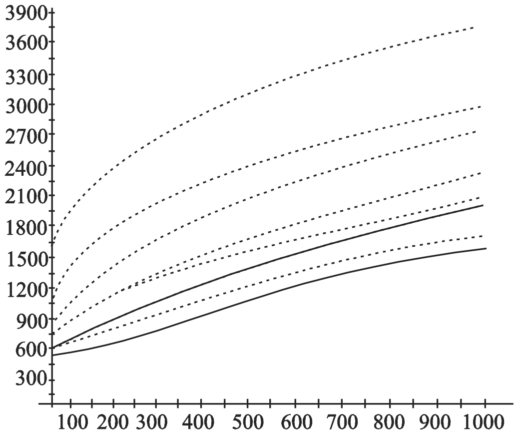
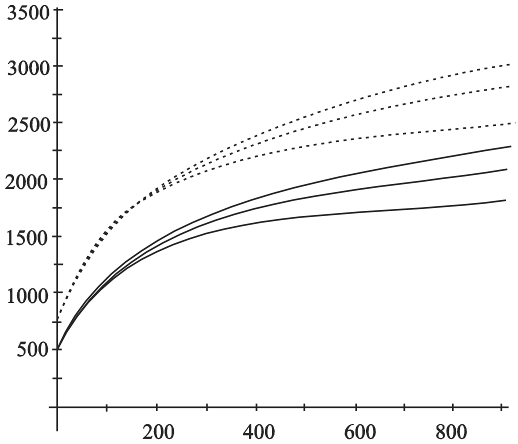
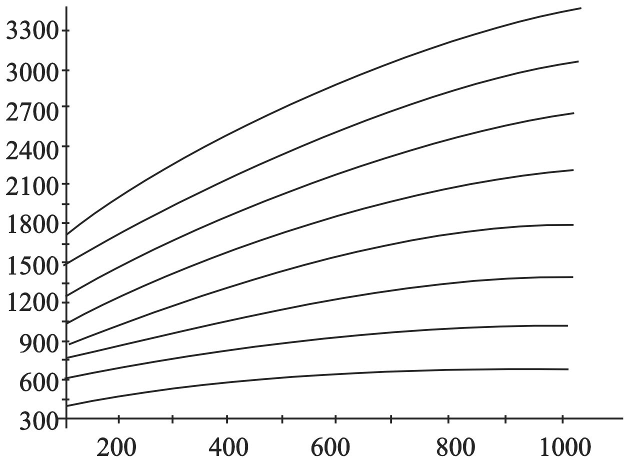


Variation of critical Rayleigh number Ra_c with H for anisotropic parameter n_s



Variation of critical Rayleigh number Ra_c with H for anisotropic parameter n_s



Variation of critical Rayleigh number with H for different values of Ta

Results and Discussions

Linear stability analysis of a fluid-saturated rotating porous cavity is carried out by considering a thermal non-equilibrium model. The variation of critical Rayleigh number based on the mean properties of the porous medium with the interphase heat transfer coefficient H for a range of values of the parameters γ, ξ, n_f, n_s and Ta is considered. In figure 1 we show that the effect of mechanical anisotropy parameter ξ on critical Rayleigh number is Ra_c for a fixed value of $\gamma = 1.0, \eta_f = 10, \eta_s = 10, T_a = 5$. From this figure it is evident that increase in the value of ξ decreases Rc and thus advances the onset of convection. This may be understood as follows: let us keep the

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vertical permeability '*h*' fixed (or the horizontal permeability 'a' fixed), and vary the horizontal permeability '*h*' (or the vertical permeability). Then an increased horizontal permeability reduces the Rayleigh number indicating that the system becomes unstable.

In figure 2 we show that critical Rayleigh number variation with H for different values of porosity modified conductivity ratio γ. We observe from this figure that for small values of H there is no transfer of heat between the phases and the onset of criterion is not affected by the properties of the solid phase. The condition for the onset of convection is based on the mean properties of the medium and therefore, the critical Rayleigh number is function of γ. In this figure for fixed values of $\eta_f = 1.0, \eta_s = 1.0, Ta = 10, \xi = 1.0$ and for small values of H, critical Rayleigh number increases with the decreasing values of γ. The effect of porosity modified conductivity ratio therefore, is to increase the stabilizing effect of both rotation and inter-phase heat transfer coefficient γ = 0.3. $\xi = 1.0, \eta_s = 1.0, Ta = 10$ With the effect of small value of Taylor's number we find that an increase in the value of η_f increases the value of Ra_c indicating that the effect of increasing the thermal anisotropy parameter is to delay the onset of convection.

Figure 3 shows the effect of thermal anisotropy parameter η_f of the fluid phase on Ra_c for $\gamma = 0.3, \xi = 1.0, \eta_s = 1.0$ and $Ta = 10$. Its effect is found to be similar to that of η_f. However, for small values of H, Ra_c is found to be independent of η_s. Figure 4 shows the effect of thermal anisotropy parameter η_s of the solid phase on R_c for γ = 0.3.

$\xi = 1.0, \eta_f = 1.0$ and Ta = 10. Its effect is found to be similar to that of η_f.

The variation of critical Rayleigh number Ra_c with interphase heat transfer coefficient H is shown in figure 5 for different parameter values Ta. For small values of Taylor number the stationary onset occurs. As the Taylor number increases the critical Rayleigh number is increased. Therefore, the rotation enhances the stability of the system in stationary modes.

Conclusions

The linear stability of a horizontal fluid-saturated porous channel with the effect of coriolis force of studied numerically when the fluid and solid phases are not in local thermal equilibrium. In case of linear stability theory we derived critical Rayleigh number as a function of Taylor number, interphase heat transfer coefficient, porosity modified conductivity ratio and mechanical anisotropy parameters. We found that there is a competition between the processes of rotation and thermal diffusion that causes convective instability, and also we found that for small value of interphase heat transfer coefficient the system behaves like a local thermal equilibrium model. The rotation has stabilizing effect on convection. The effect of porosity modified conductivity ratio is to advance the onset of convection, while the diffusivity ratio strengths the stabilizing effect of rotation and interphase heat transfer coefficient. For small value of H, the critical values are independent of the porosity modified conductivity ratio.

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