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An Application of Fractional Calculus in RLC Electrical Circuit Using Shehu Transform

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Abstract: In this paper fractional differential equation of RLC electrical circuit has been discussed . The solution of this differential equation is unique in nature. A series of papers [2,10] already discussed the solution of fractional differential equation on RLC circuit by Laplace transform technique. We apply Shehu transform for solving the fractional differential equation of RLC electrical circuit. This transform technique gives an additional variable which may be benefited to scientist who wants to apply RLC circuit in two variables. This is an extended result of Ali's et.al.[10]

Keywords: RLC circuit, Fractional derivative, Shehu transform, Inverse Shehu transform.

Mathematics subject classification 2010 No.: 34A08, 35A22, 33E12, 35C10.

1. Introduction: First we will introduce the calculus, which was discovered by Isaac Newton and Gottfried Wilhelm in seventeenth century. Calculus is a type of Mathematics which is deal with rate of change, for example the velocity of object and the acceleration of object. When Leibnitz introduced the calculus in 1695 at that time L' hospital asked a query to him what will happened if n will take $1/2$ value in $\frac{d^n y}{dx^n}$ .After two year in 1697 Leibnitz replied him by wrote a letter "One day useful consequence will be drawn from this fact" The credit of organizing first international conference on fractional calculus goes to Bertram Ross was held in New Heaven(U.S.A) in 1974. After that more than three hundred years have been passed with these passing years, several aspects of fractional calculus have been developed and studied. In other words, the fractional calculus operator deal with integrals and derivatives of arbitrary order.

Mathematicians and Physicists found that the fractional calculus operators is very useful in a various fields such as quantitative biology, electro chemistry, scattering theory, transport theory, probability, elasticity, control theory, potential theory, signal processing, image processing, diffusion theory, kinetic theory, heat transfer theory and circuit theory etc.

The accurate use of a derivative of non-integer order is due to the French mathematician S. F. Lacroix [23] in 1819.He expressed the derivative of non-integer order ½ in terms of Legendre's factorial symbol Γ.

$$\Gamma(a) = \int_0^\infty t^{\alpha-1} e^{-t} dt$$

Starting, with a function $y = x^m$, Lacroix expressed it as follow

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$$\frac{d^n y}{dx^n} = \frac{m!}{(m-n)!} x^{m-n} = \frac{\Gamma(m+1)}{\Gamma(m-n+1)} x^{m-n}$$

Replacing nwith$\frac{1}{2}$ and putting $m = 1$, he obtained the derivative of order $\frac{1}{2}$ of the function x.

$$\frac{d^{1/2} y}{dx^{1/2}} = \frac{\Gamma(2)}{\Gamma(3/2)} x^{1/2} = \frac{2}{\sqrt{\pi}} \sqrt{x}$$

In 1822, J. B. J. Fourier made the following integral representation

$$\frac{d^u f(x)}{dx^u} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(\alpha) d\alpha \int_{-\infty}^{+\infty} p^u \cos\left(px - p\alpha + \frac{u\pi}{2}\right) dp$$

where the number u was regarded as any quantity whatever, positive or negative.

The credit of first application of fractional calculus goes to Abel's [11], he employed it in the solution of an integral equation, which emerged in the formulation of the tautochroneproblem of finding the shape of a frictionless wire lying in a vertical plane such that the time of slide of a bead placed on the wire to the lowest point of the wire is the same regardless of position of the bead on the wire.

2. Definitions and Preliminaries There are several definitions of a fractional derivative of order $\alpha \geq 0$ (see Kilbas et al. (2006), Podlubny (1999)). The most commonly used definitions are the Riemann-Liouville and Caputo. In this section, we give some basic definitions .

The Mittag-Leffler Function:

The Mittag-Leffler function introduced by Mittag-Leffler [5] in 1903, is defined as

$$E_\alpha(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k + 1)}, (\alpha \in C, Re(\alpha) > 0) \quad (1)$$

Generalization of the Mittag-Leffler function is given by Wiman [3] in 1905, defined as

$$E_{\alpha,\beta}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k+\beta)}, (\alpha, \beta \in C, Re(\alpha) > 0, Re(\beta) > 0) \quad (2)$$

Prabhakar [24] introduced generalization of Mittag-Leffler function in 1971 in the form

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$$E_{\alpha,\beta}^{\gamma}(x) = \sum_{k=0}^{\infty} \frac{(\gamma)_k x^k}{\Gamma(\alpha k + \beta)\, k!},$$

$(\alpha, \beta, \gamma \in C, Re(\alpha) > 0), Re(\beta) > 0$ (3)

Where $(\gamma)_k$ is the Pochammer symbol

It is an entire function with $\rho = [Re(v)]^{-1}$.

For$\gamma = 1$, this function coincides with(2),while for $\gamma = \beta = 1$with

$$E_{\alpha,\beta}^{1}(x) = E_{\alpha,\beta}(x), E_{\alpha,1}^{1}(x) = E_{\alpha}(x) \tag{4}$$

We also have

$$\phi(\beta, \gamma; x) = {}_1F_1(\beta, \gamma; x) = \Gamma(\gamma)\, E_{1,\gamma}^{\beta}(x) \tag{5}$$

$$E_{1,\gamma}^{\beta}(x) = \frac{1}{\Gamma(\gamma)} H_{1,2}^{1,1}\left[-x \middle| \begin{matrix} (1-\gamma, 1) \\ (0,1), (1-\beta, \alpha) \end{matrix}\right], Re(\alpha) > 0; \alpha, \beta, \gamma \in C \tag{6}$$

For $\gamma = 1(6)$ gives rise to the following result for the generalized Mittag-Leffler function.

$$E_{\alpha,\beta}(x) = H_{1,2}^{1,1}\left[-x \middle| \begin{matrix} (0, 1) \\ (0,1), (1-\beta, \alpha) \end{matrix}\right], Re(\alpha) > 0; \alpha, \beta \in C \tag{7}$$

If we further take $\beta = 1$in (7)we find that

$$E_{\alpha}(x) = H_{1,2}^{1,1}\left[-x \middle| \begin{matrix} (0, 1) \\ (0,1), (0, \alpha) \end{matrix}\right], Re(\alpha) > 0; \alpha \in C \tag{8}$$

Several mathematicians have contributed for developing the fractional calculus by giving the fractional derivative such as Riemann- Liouville operator, Modified Reimann- Liouville fractional derivative, Caputo fractional derivative, Wely fractional operator, Tuan and Saigo Fractional Operators these all mathematicians have given the definition of fractional derivative . The Reimann –Liouville fractional derivative of constant and Caputo fractional derivative of constant are not equal to each other .The Caputo

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fractional derivative of constant is zero. So Caputo derivative is more useful for applications in science and engineering.

3. Caputo Fractional Derivative:

The Caputo fractional derivative of order $\alpha > 0$ is introduced by Caputo [8] in the form

(if $m-1 < \alpha \leq m, Re(\alpha) > 0, m \in N$)

${}_a^cD_t^\alpha f(t) = I^{m-a} D^m$ f(t)

or

$${}_a^cD_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^m(\tau)d\tau}{(t-\tau)^{\alpha+1-m}} \qquad ,t>0$$

$$= \frac{d^m f(t)}{dt^m}, \text{if} \alpha = m \qquad (9)$$

where $\frac{d^m f(t)}{dt^m}$ is the m-th derivative of order m of the function $f(t)$ with respect to t.

or

$${}_0^cD_x^\alpha f(x) = \frac{1}{\Gamma(1-\alpha)} \int_0^x \frac{f'^{(t)}}{(x-t)^\alpha} dt,$$

$$\text{where } 0 < \alpha < 1) \qquad (10)$$

According to this definition

${}_a^cD_t^\alpha A = 0$ $\qquad$ f(t) = A = Constant

This is Caputo's fractional derivative of a constant is zero. The Shehu transform of Caputo derivative is representation of

$$S[{}_a^cD_t^\alpha f(t)] = \frac{s^\alpha}{u^\beta} F(s, u) - \sum_{k=0}^{n-1} (\tfrac{s}{k})^{\alpha-(k-1)} \ [D^k f(t)]_{t=0} \qquad (11)$$

We see that from the equation (11) the representation of the Caputo derivative in Shehu domain using the initial condition $D^k(0)$ where k is integer. When the initial condition is zero then the equation (11) converted into

$S[{}_a^cD_t^\alpha f(t)] = (\frac{s}{u})^\alpha F(s)$

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Unit Ramp Function:

Let $f(t)$ be the unit ramp function [9], which can be mathematically expressed as follows:

$$f(t) = \begin{cases} 0 & for\ t < 0 \\ t & for\ t \geq 0 \end{cases} \quad (12)$$

And its Shehu transform is $\frac{u^2}{s^2}$.

Unit Parabolic Function:

The unit parabolic function [9] defined as follows:

$$f(t) = \begin{cases} 0 & for\ t < 0 \\ \frac{t^2}{2} & for\ t \geq 0 \end{cases} \quad (13)$$

And its Shehu transform is $\frac{u^3}{s^3}$.

4. Shehu Transform:

Shehu transform is Laplace-type integral transform which is used for solving ordinary and partial differential equations. Shehu transform is converge to Laplace transform when u=1,and to Yang integral transform when s=1.Shehu transform is denoted by an operator S[.].

$$S[i_C(t)] = F(s,u) = \int_0^\infty e^{\frac{-st}{u}} i_C(t)\, dt$$

5. *RLC* Electrical Circuit:

In this section, we have considered a RLC electrical circuit with a capacitor and an inductor, connected in parallel and this set is connected in series with a resistor and a voltage. The capacitance *C*, the inductance *L* and the resistor *R* are consider positive constants and $\psi(t)$ is the unit ramp function. Recently, Soubhia, Camargo and Rubens [2], consider the $\psi(t)$ is Heaviside function in their paper.

The following equations associated with a three elements of *RLC* electrical circuit as under:

The voltage drop

$$U_L(t) = L\frac{d}{dx}I(t), \qquad \text{across an inducter;}$$

The voltage drop

$$U_c(t) = \frac{1}{C}\int_0^t I(\xi)d\xi, \text{across a capacitor}$$

The voltage drop

$$U_c(t) = \frac{1}{C}\int_0^t I(\xi)d\xi, \text{across a capacitor}$$

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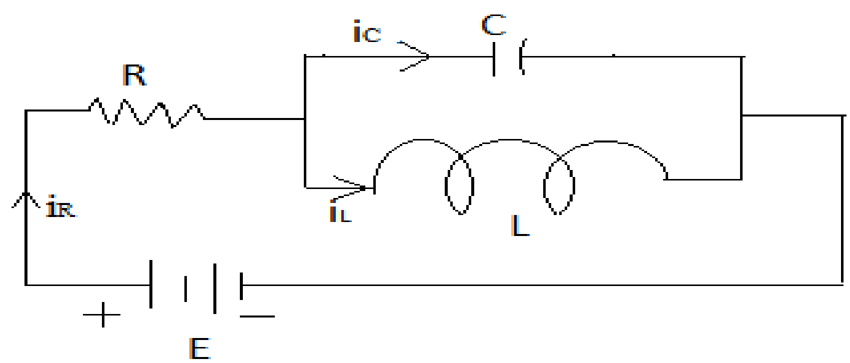
where$I(t)$ is the current.

Applying the Kirchhoff's voltage law and above equations associated with the three elements. The non-homogeneous second order ordinary differential equation can be written as

$$RC\frac{d^2}{dt^2}U_C(t)+\frac{d}{dt}U_C(t)+\frac{R}{L}U_C(t)=\frac{d}{dt}\psi(t) \quad (14)$$

Here, $U_c(t)$ is the voltage in the capacitor, this is the same on the inductor as we can see in following figure (1), because they are connected in parallel.



Three element LCR electrical circuit

Fig-1

Again, for current on the inductor, we get other non-homogeneous second order ordinary differential equations,

$$RLC\frac{d^2}{dt^2}i_L(t)+L\frac{d}{dt}i_L(t)+Ri_L(t)=\psi(t) \quad (15)$$

Now, using the above voltage drop equation across a capacitor for the inductor, therefore these two non-homogeneous second order ordinary differential equations correspond to integro-differential equations as follows.

$$R\frac{d}{dt}i_C(t)+\frac{1}{C}i_C(t)+\frac{R}{LC}\int_0^t i_C(\xi)d\xi,=\frac{d}{dt}\quad\psi(t) \quad (16)$$

And

$$RC\frac{d}{dt}U_L(t)+U_L(t)+\frac{R}{L}\int_0^t U_L(\xi)d\xi,=\quad\psi(t) \quad (17)$$

We observe that, both integro-differential equations are in the similar form. Here we consider only the first one i.e. (18). The above integro-differential equation (16) solved by technique of Shehu transform with

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initial condition $i_C(0) = 0$ in classical methodology and the solution found in terms of an exponential function [7].

6. Fractional Integro-Differential Equation:

In this, we generalize ordinary differential equations (16) into the fractional form associated with a current on the capacitor, which is known as fractional integro-differential equation:

$$R\frac{d^\alpha}{dt^\alpha}i_C(t) + \frac{1}{C}i_C(t) + \frac{R}{LC}\frac{1}{\Gamma(\alpha)}\int_0^t (t-\xi)^{\alpha-1} i_C(\xi)d\xi = \frac{d}{dt}\psi(t) \qquad (18)$$

With $0 < \alpha \leq 1$, and the fractional derivative is taken in the Caputo sense, where $\psi(t)$ is the unit ramp function. In this case, $i_C(t)$can be interpreted as a Green's function because the second member is unit ramp function. Let us consider $i_C(0) = 0$, i.e., the initial current on the capacitor is zero. We obtained the result of equation (18) is same as fractional integro-differential equation(20) associated with the RLC electrical circuit, if $\alpha = 1$. This replacement can be useful in discussing the corresponding numerical problem, for a particular value of the parameter, because the solution is shown in terms of a closed expression.

Use the Shehu integral transform[22] to solve this fractional integro-differential equation:

$$S[i_C(t)] = F(s,u) = \int_0^\infty e^{\frac{-st}{u}} i_C(t)\,dt$$

with $Re(s) > 0$, and the following algebraic equation,

$$R(\frac{s}{u})^\alpha F(s,u) + \frac{F(s,u)}{C} + \frac{R}{LC}F(s,u)(\frac{u}{s})^\alpha = \frac{u}{s}$$

The solution is given by

$$F(s,u)R\,\frac{(s^{2\alpha}+bu^{2\alpha}+as^\alpha u^\alpha)}{u^\alpha s^\alpha} = \frac{u}{s}$$

$$F(s,u) = \frac{1}{R}\frac{u^{2\alpha+1}s^{\alpha-1}}{s^{2\alpha}u^\alpha + au^{2\alpha}s^\alpha + bu^{3\alpha}}$$

Where, $a = 1/RC$ and $b = 1/LC$ are the positive parameters. If $\alpha \geq \beta, \alpha > \gamma, a \in R$ $|A| < (\frac{s}{u})^\alpha$,and $|B| < \frac{s^\alpha u^\beta + as^\beta u^\alpha}{u^{\alpha+\beta}}$

To recover the solution of the fractional integro-differential equation, we proceed with the inverse Shehu transform [1]

$$i_C(t) = \frac{1}{R}S^{-1}[\frac{u^{2\alpha+1}s^{\alpha-1}}{s^{2\alpha}u^\alpha + au^{2\alpha}s^\alpha + bu^{3\alpha}}]$$

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Using the relation

$$S^{-1}\left[\frac{u^{\alpha+\beta-\gamma}s^{\gamma}}{s^{\alpha}u^{\beta}+Au^{\alpha}s^{\beta}+Bu^{\alpha+\beta}}\right]=t^{\alpha-\gamma-1}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-B)^{n}(-A)^{k}\binom{n+k}{k}}{\Gamma((k(\alpha-\beta)+(n+1)\alpha-\gamma)}t^{k(\alpha-\beta)+n\alpha}$$

$$\text{Valid for }\left|\frac{-Bu^{\alpha+\beta}}{s^{\alpha}u^{\beta}+Au^{\alpha}s^{\beta}}\right|<1\text{ and }\alpha\geq\beta,$$

We can write

$$i_{C}(t)=\frac{t^{\alpha}}{R}\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-b)^{n}(-a)^{k}\binom{n+k}{k}}{\Gamma((k(\alpha)+(n+1)\alpha+1)}t^{k(\alpha)+n2\alpha} \tag{19}$$

.

This completes the analysis.

7 Conclusion: In this paper, we have applied Shehu transform to find the solution of non homogenous fractional differential equation of RLC circuit, which is obtained in terms of two variables double series it will be help to electrical engineers for working on any model of two variables which was not discussed by Ali et.al. and we have obtained unique result. This work may be an open research problem for electrical engineers in future. One day useful consequences will be drown from our result.

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