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THE FORCING DETOUR DOMINATION DEGREE NUMBER OF A GRAPH

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ABSTRACT

Let G be a connected graph and S is a minimum detour dominating degree set of G.A subset T⊆S is called a forcing subset of S if S is the unique minimum detour dominating degree set containing T.A forcing subset for S of minimum cardinality is a minimum forcing subset of S .The forcing detour dominating degree number of S, denoted by $f_{\gamma d}^{d}(G)$,is the cardinality of a minimum forcing subset of S but the forcing detour domination degree number denoted $f_{\gamma d}^{d}(G)$ $=\min\{ f_{\gamma d}^{d}(G)\} = \max\{l^{d}(S)\}=\max\{deg(u)+deg(v)+\sum deg(w) \}$where the minimum is taken over all minimum detour dominating degree set S in G but in the mini $f_{\gamma d}^{d}(S)$ we find the $\max\{l^{d}(G)\}$ in G.

Introduction

In this paper, we introduce the new concept forcing detour domination degree number of a graph. For two vertices u and v in a graph G = (V,E), the detour distance D(u, v) is the length of a longest u—v path in G. A u—v path of length D(u, v) is called a u—v detour. A set S ⊆ V is called a detour set of G if every vertex in G lies on a detour joining a pair of vertices of S. The detour number dn(G) of G is the minimum order of its detour sets and any detour set of order dn(G) is a detour basis of G.The concept of forcing refer to a situation where the inclusion of a particular vertex in a dominating set compels the inclusion of certain other vertices in the set due to the structure of the graph.The forcing detour domination degree number involves two important ideas detour domination and forcing domination. It measures the number of vertices whose inclusion in a dominating set is "forced" by the detour paths in the graph.A vertex is said to "force" other vertices into the dominating set if those vertices would not be dominated unless a detour path through the forced vertex is considered. The forcing detour domination degree number quantifies how many vertices are indirectly dominated through detours and how many vertices must be included in the dominating set to ensure full domination of the graph under the detour domination criteria.

This concept is particularly useful in understanding how certain vertices in the network play a critical role in ensuring that the entire system is connected or dominated,even when direct connections are not available.The forcing detour domination degree number is applicable in scenarios such as Network design ,robustness analysis and strategic planning where indirect connections can significantly impact the overall system behaviour.

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Keywords:

Domination, Detour domination, Detour degree number, Detour number, forcing detour degree number.

Defnition 3.1

Let G be a connected graph and S is a minimum detour dominating degree set of G.A subset T⊆S is called a forcing subset of S if S is the unique minimum detour dominating degree set containing T.A forcing subset for S of minimum cardinality is a minimum forcing subset of S .The forcing detour dominating degree number of S, denoted by $f_{\gamma d}{}^{d}(G)$,is the cardinality of a minimum forcing subset of S but the forcing detour domination degree number denoted $f_{\gamma d}{}^{d}(G)$ = mini{ $f^{d}(G)_{\gamma d}$= max{$l^{d}(S)$}=max{deg(u)+deg(v)+∑deg(w) }where the minimum is taken over all minimum detour dominating degree set S in G but in the mini $f_{\gamma d}{}^{d}(S)$ we find the max{$I^{d}(G)$} in G.

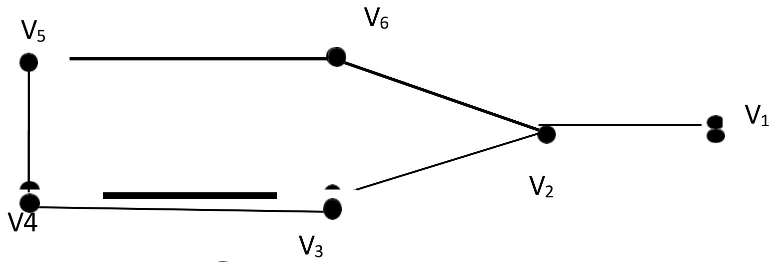

Figure 3(a)

Example 3.2

For the graph G given in figure 3(a) $S_1=\{v_1,v_3,v_6\}$, $S_2=\{v_1,v_4,v_6\}$,$S_3=\{v_1,v_5,v_3\}$are the only three detour dominating set of G such that $f_{\gamma d}{}^{d}(S_1) = f_{\gamma d}{}^{d}(S_2) = f_{\gamma d}{}^{d}(S_3)=2$ so that max $l^{d}(s_1) = deg(v_1)+deg(v_6)+$ the sums taken over all the intermediate vertices or $\sum deg(w)$.So that $\max\{l^{d}(s_1)\} = deg(v_1)+deg(v_6)+deg(v_2)+deg(v_3)+deg(v_4)+deg(v_5) = 1+2+3+2+2+2 = 12$.

Similarly $\max\{l^{d}(s_2)\} = \max\{l^{d}(s_3)\} = \max\{12,12,12\} = 12$.So that $f_{\gamma d}{}^{d}(u,v)=12 = 2p$.

Theorem:3.3

For every connected graph G, $0\le f_{\gamma d}{}^{d}(G)\le \gamma^{d}{}_{d}(G)$

Proof:

By definition of forcing detour dominating degree set, $f_{\gamma d}{}^{d}(G)\ge 0$.Since every detour dominating degree set have a forcing subset, $f_{\gamma d}{}^{d}\le \gamma^{d}{}_{d}(G)$.Thus $0\le f_{\gamma d}{}^{d}\le \gamma^{d}{}_{d}(G)$.

Observation:3.4

For any connected graph G,$2\le \max\{d_n(G),\gamma(G)\}\le \gamma_d(G)$.

Theorem:3.4

Let a connected graph be G .Then

i) $f_{\gamma d}^{d}(G) = 0$ iff G has a unique minimum detour dominating degree set.
ii) $f_{\gamma d}^{d}(G) = 1$ iff G has atleast two minimum detour dominating degree sets,one of which is a unique minimum detour dominating degree set containing one of its elements and
iii) $f_{\gamma d}^{d}(G)=\gamma^{d}_{d}(G)$ iff no minimum detour dominating degree set of G is the unique minimum detour dominating degree set containing any of its proper subsets.

Proof:

i) Let $f_{\gamma d}^{d}(G) = 0$ Then by definition 3.1, $f_{\gamma d}^{d}(G) = 0$ for some minimum detour dominating degree set of G.So that the empty set φ is the minimum furcing subset for S.Since the empty set φ is a subset of every set,it follows that S is a unique minimum detour dominating degree set of G.The converse of the theorem is obvious.

ii) Let $f_{\gamma d}^{d}(G) = 1$ .Then by Theorem 3.4(i) g has atleast two minimum detour dominating degree sets.Also find $f_{\gamma d}^{d}(G)=1$ there is singleton subset T of a minimum detour dominating degree set S of G such that T is not a subset of any
other minimum detour dominating degree set of G.Thus S is a unique minimum detour dominating degree set containing one of its element.The converse is obvious.

iii) Let $f_{\gamma d}^{d}(G)=\gamma^{d}_{d}(G)$.Then $f_{\gamma d}^{d}(G)=\gamma^{d}_{d}(G)$ for every two minimum detour dominating degree sets S of G.Also by observation 3.4 $\gamma^{d}_{d}(G) \geq 2$.Then by Theorem 3.4(a),G has atleast two minimum detour dominating degree set and so that empty set φ is not a forcing subset for any minimum detour dominating degree set s of G. Since $S_1=\{V_1, V_2, V_3\},)=\gamma^{d}_{d}(G)$,no proper subset of S is a forcing subset of S.
Thus no minimum detour dominating degree sets containing any one of its proper subsets. Conversly the data implies that G contains more than one minimum detour dominating degree set S other than S is a forcing subset of S.Hence it follows that $f_{\gamma d}^{d}(G)=\gamma^{d}_{d}(G)$.

Definition:3.5

A vertex V is said to be detour dominating degree vertex of G if V belongs to every minimum detour dominating degree set of G.

Example :3.6

For a graph G given in fig3(a) $S_1=\{V_1, V_2, V_3\}$, $S_2=\{V_1, V_4, V_6\}$, $S_3=\{V_1, V_5, V_6\}$are the only 3 detour dominating degree set of G.So that $f_{\gamma d}^{d}(G)=2$.It is clear that the detour dominating degree vertex of G.

Theorem:2.4

Every end vertices of G belongs to every detour set of G.

Theorem:3.7

For a star $G = K_{1,p-1}$, $f_{\gamma d}^{d}(G) = 0$

Let G = $K_{1,p-1}$ with center vertex V.

Let S be the set of all end vertices of G. Then by theorem (2.4) S = V(G)-{V}is the unique

γ^d_d set of G so that $f_{\gamma d}{}^d(G) = 0$.

Theorem:3.8

For a double star $f_{\gamma d}{}^d(G) = 0$.

The proof of the theorem is similar to that of theorem 3.7

$f_{\gamma d}{}^d$(G)For a complete graph G = K_p (p≥3), $f_{\gamma d}{}^d(G) = 2$.

Let x,y be two vertices of the graph G.Then S= {x,y}is a γ^d_d – set of G so that

$\gamma^d_d(G) = 2$.Since p≥3, γ^d_d – set is not unique and $f_{\gamma d}{}^d(G)\geq 1$. Suppose that $f_{\gamma d}{}^d(G) = 1$,then there exist x∊G such that x belongs to only one γ^d_d – set of G. Since p≥3,there are atleast three γ^d_d – set which implies that x is not adjacent to atleast one vertex of G which is contradiction to G is complete. Therefore $f_{\gamma d}{}^d(G) = 2$.

Theorem :3.10

For complete bipartite graph G= $K_{m,n}$ $f_{\gamma d}{}^d(G) = \begin{cases} 2 \ if \ m = 3 \\ 2 \ if \ m = 2, n \geq 3 \\ 3 \ if \ m < n \end{cases}$

Proof:

Let U and V be the two bipartitite set of G

Let U = $\{u_1,u_2 \ldots\ldots\ldots\ldots u_m\}$ and W = $\{w_1,w_2,\ldots\ldots\ldots w_m\}$

Case (i)

If m= n then $S_{ij} = \{u_i,w_j\}$,$(1\leq i \leq m, 1 \leq j \leq n)$ is a minimum detour domination degree set of G such that $f_{\gamma d}{}^d(S_{ij}) = 2$,$(1\leq i \leq m, 1 \leq j \leq n)$ .Therefore $f_{\gamma d}{}^d(S_{ij}) = 2$.

Case(ii)

If m =2 ,n≥ 3 then $S_{ij} = \{u_i,w_j\},\}$,$(1\leq i \leq m, 1 \leq j \leq n)$ is a minimum detour domination degree set of G such that $f_{\gamma d}{}^d(S_{ij}) = 2$,$(1\leq i \leq m, 1 \leq j \leq n)$ .Therefore $f_{\gamma d}{}^d(S_{ij}) = 2$.

Case (iii)

If m< n $S_{ij} = \{u_i,u_j,w_k\}$,$(1\leq i \leq m, 1 \leq j \leq n, i \neq j, i \leq k \leq n)$ is a minimum detour domination degree set of G such that $f_{\gamma d}{}^d(S_{ijk}) = 3 1\leq i \leq m, 1 \leq j \leq n, i \neq j, i \leq k \leq n)$.Therefore $f_{\gamma d}{}^d(S_{ij}) = 3$.

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